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by the Chairman

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Mathematical Tables in Reports of the B.A.A.S.

For over seventy consecutive years the British Association for the Advancement of Science has had a Committee on Calculation of Mathematical Tables, and we hope to be in a position later to present a detailed account of its membership and activities. Meanwhile we list chronologically the Mathematical Tables published (1877–1929) in the *Reports* of the B.A.A.S., and a few others related to them, which appeared elsewhere. Practically all of these Tables are by members of the Committee and most of them are in the field of Bessel Functions (1889–1928). There were also some tables in J. W. L. Glaisher's article, "On the law of distribution of prime numbers," *1872* (Transactions, p. 20).

1873 (p. 1–175).—Before any Tables were published there appeared the remarkable **1**. ***Report of the Committee on Mathematical Tables*** by J. W. L. Glaisher. This is a sort of catalogue and analysis of Mathematical Tables up to that time. It was an elaboration of the idea illustrated by Augustus De Morgan's article on "Mathematical Tables" in ***The English Cyclopaedia, Arts and Science Section***, v. 7, 1861, cols. 976–1016. Important supplements to Glaisher's ***Report***, by A. Cayley and others, were published in 1875, 1878, 1879, 1880, 1881, and 1883.

1877 (Notices and abstracts, p. 8–14).—**1A** [added in proof]. J. C. Adams, "On the calculation of Bernoulli's numbers up to B_{62} by means of Staudt's theorem," with tables (not published by the Committee) expressing the numbers in (a) vulgar fractions, (b) repeating decimals. The first table was printed also in ***Jn. f. d. reine u. angew. Math.***, v. 85, 1878, p. 270–271; and the second is given in ***Scientific Papers*** of Adams, v. 1, Cambridge, 1896, p. 455–458.

1879 (p. 49–57 + plate I).—**2**. J. W. L. Glaisher, "Legendrian functions," calculated in 1872. $P_n(x)$, for $x = [0.00(0.01)1; 2D \text{ to } 18D], n = 1(1)7$. A listing of a table by Samuel Houghton for calculation of sun-heat coefficients (p. 66–71) is omitted.

1889 (p. 28–32).—**3**. A. Lodge, $I_n(x)$ for $x = [0.0(0.2)6.0; 12S], n = 0(1)11$, the last figure approximate. $I_n(x) = i^{-n}J_n(ix)$. Compare nos. **5**, **6**.

1893 (p. 82–119).—**4**. C. E. Bickmore, "Tables connected with the Pellian equation $[y^2 = ax^2 - 1]$ from the point where the work was left by Degen in 1817"; report by A. Cayley (p. 73–120) also in ***Collected Math. Papers***, v. 13, 1897, p. 430–467. Extends the table from $a = 1001$ to $a = 1500$.

1893 (p. 227–279).—**5**. A. Lodge, $I_1(x)$ for $x = [0.001(0.001)5.100; 9D]$, last figure approximate. With differences. On p. 228 there is a table of $J_0(x\sqrt{i})$ for $x = [0.0(0.2)6.0; 9D]$. Compare nos. **3**, **6**.

1896 (p. 98–149).—**6**. A. Lodge, $I_0(x)$ for $x = [0.000(0.001)5.100; 9D]$, last figure approximate. With differences. It was found that nos. **5** and **6** were "subject to errors of two or three units in the last decimal place"; see notes on Table VI, p. xv, of B.A.A.S., ***Mathematical Tables, v. 6, Bessel Functions***, part I, Cambridge, 1937. Compare nos. **3**, **5**. It was at this time that the Committee recommended the publication of a volume of tables of Bessel functions to 6D.

1899 (p. 65–120).—**7.** ALICE LEE, $G(r, \nu)$-integrals, for $\phi = 0°(1°)45°$, $r = 1(1)50$, where $\tan\phi = \nu/r$. $G(r, \nu) = \int_0^\pi \sin^r \theta e^{\nu\theta} d\theta = e^{(1/2)\nu\pi} F(r, \nu)$ $= [(e^{\nu\phi + (1/2)\nu\pi}(\cos\phi)^{r+1}/\sqrt{r-1})]H(r, \nu)$. The tables are to 7D, with first and second differences, of $\log F(r, \nu)$ and $\log H(r, \nu)$. These are in a report of Karl Pearson, and not of the Mathematical Tables Committee. They are reprinted in Table LIV (p. lxxxi–lxxxiii, 126–142) of *Tables for Statisticians and Biometricians*, ed. K. Pearson, part I, London, 1914; second ed., 1924; third ed. 1930.

1907 (p. 94–97 + folding plate).—**8.** A. LODGE, Table I: $\log R_n$ to 8D; Table II: $\log [R_n(2/\pi)^{1/2}]$ to 7D; Table III: $Q_0(x)$; 10D, $\log [-Q_0(x)]$; 8D, $\log(-\sin\alpha_0)$; 8D, $-\sin\alpha_0$; 9D, $\log(-x\sin\alpha_0)$; 8D, all for $x = 10(10)100(100)1000$, and $n = 0(1/2)(13/2)$, where $J_n(x) = (2/\pi x)^{1/2}R_n(x)\cos\left(x + \alpha - \frac{\pi}{4} - n\frac{\pi}{4}\right)$, $R_n^2 = P_n^2 + Q_n^2$, $\sin\alpha = Q_n/R_n$, $P_n(x) = 1 - \frac{(4n^2-1)(4n^2-3^2)}{1\cdot 2(8x)^2} + \frac{(4n^2-1)\cdots(4n^2-7^2)}{1\cdot 2\cdot 3\cdot 4(8x)^4} - \cdots$; $Q_n(x) = \frac{4n^2-1}{8x} - \frac{(4n^2-1)(4n^2-3^2)(4n^2-5^2)}{1\cdot 2\cdot 3(8x)^3} + \cdots$. Concluded in no. **9**.

1909 (p. 33–36).—**9.** A. LODGE, Table IV: $Q_i(x)$, $i = 0, 1, \cdots, 6$; $x = [10(10)100(1000; 9D–10D]$. Table V: $\log [8x\alpha/(4n^2-1)]$, $n = 0(1/2)(13/2)$; $x = [10(10)100(100)1000; 7D]$, "last figure approximative." Varying values for α. Continued from no. **8**.

1911 (p. 67–71).—**10.** G. GREENHILL, Specimen tables of the elliptic function for modular angle $\theta = 45°$ and $\theta = 75°$, the argument proceeding by degrees of the quadrant of the quarter period K or F (90°).

1911 (p. 73–78).—**11.** J. R. AIREY, Neumann functions $G_n(x)$ and $Y_n(x)$, i.e., $G_0(x)$, $G_1(x)$, $Y_0(x)$, $Y_1(x)$, for $x = [0.0(0.1)16.0; 7D]$, the greatest error being 10^{-7}. There is a reference to W. S. ALDIS, R. So. London, *Proc.*, v. 66, 1900, p. 41, for a table of $G_0(x)$ and $G_1(x)$ for $x = [0.0(0.1)6.0; 21D]$. Compare nos. **15, 16**.

1912 (p. 39–55).—**12.** G. GREENHILL, Four specimal tables of elliptic integrals, calculated for the modular angles $\theta = 15°$, $K' = K\sqrt{3}$; 45°, $K' = K$; 75°; and 80°1, $K = 2K'$.

1912 (p. 56–68).—**13.** A. G. WEBSTER, Ber x, bei x, and their derivatives, $x = [0.0(0.1)10.0; 9D]$, with 7 differences. "The last 35 entries for the derivative of bei(x) have been corrected to 8D." (Committee's report, 1929). Published because of "especial service to electrical engineering." H. G. SAVIDGE in no. **25** noted that there was an error in Webster's table of "bei x which necessitated the recalculation of part of the table. The error became considerable as the argument increases, and the corrected figures" from $x = 6.5$ to 10.5 are given *1916*, p. 122. These are the 41, not "35 items."

1913 (p. 88–113 + 4 plates).—**14.** G. GREENHILL, Elliptic functions, for which the ratio of the periods $K/K' = 1/\sqrt{2}$, $\sqrt{2}$, $2\sqrt{2}$, $2\sqrt{3}$, 3, $3/\sqrt{2}$, $3\sqrt{2}$ $3\sqrt{3}$, 4, 5; various graphs including those corresponding to these ratios.

1913 (p. 115–130).—**15.** J. R. AIREY, Neumann functions $G_0(x)$ and $G_1(x)$, or Bessel functions of the second kind, for $x = [0.00(0.01)16.00; 7D]$. Compare nos. **11, 16**.

1914 (p. 75–82).—**16**. J. R. Airey, Neumann functions, $Y_0(x)$, $Y_1(x)$, for x = [0.02(0.02)15.50; 6D], correct to 6S. Compare nos. **11**, **17**.

1914 (p. 83–86).—**17**. J. R. Airey, $G_n(x)$ for x = [0.0(0.1)6.0(0.5)16.0; 5D], and n ranging irregularly for 3 to 14 values. Compare nos. **16**, **19**.

1914 (p. 87–102).—**18**. A. T. Doodson, Bessel functions of half integral order, $S_n(x) = (\pi x/2)^{1/2}J_{n+1/2}(x)$, $C_n(x) = (-1)^n(\pi x/2)^{1/2}J_{-n-1/2}(x)$, $S_n' \equiv dS_n(x)/dx$, $C_n'(x) \equiv dC_n(x)dx/dx$, $S_n^2 + C_n^2$, $S_n'^2 + C_n'^2$, and their common logarithms, for x = [1(1)10; 7S], and n ranging irregularly for 5 to 24 values. Compare no. **24**.

1915 (p. 27–36).—**19**. J. R. Airey, Table I: $J_n(x)$, of integral order, for x = [0.2(0.2)6.0(0.5)16.0; 6D to 10D], varying values of n. Table II: $G_0(x)$, for x = [6.5(0.5)15.5; 10D]; $Y_0(x)$, $Y_1(x)$ for x = [0.1(0.1)6.0(0.5)15.5; 10D]. Table III: $Y_n(x)$ for x = [0.2(0.2)8.0(0.5)15.5; 6D], for varying n. Compare no. **11**.

1915 (p. 36–38).—**20**. H. G. Savidge, Ker, kei, and their first derivatives, for x = [0.0(0.1)10.0; 7D–9D].

1916 (p. 59–89).—**21**. J. R. Airey, Table I: Sines and cosines for x = [0^r.000(0^r.001)1^r.600; 11D]; Table II: Table of $x - \sin x$ and $1 - \cos x$, with first differences, for x = [0^r.00001(0^r.00001)0^r.00100; 11D]. Compare no. **22**.

1916 (p. 90–91).—**22**. A. T. Doodson, Table III: Sin x, cos x, for x = [0.0(0.1)10.0; 15D]. Second editions of nos. **21** and **22** appeared in B.A.A.S., *Mathematical Tables*, v. 1, London, 1931, p. 8–23, 3.

1916 (p. 92–96).—**23**. J. R. Airey, Bessel and Neumann functions of equal order and argument, $J_n(x)$, $J_{n-1}(x)$, $G_n(x)$, $G_{n-1}(x)$, $-Y_n(x)$, $-Y_{n-1}(x)$, for n = [1(1)50(5)100(10)200(20)400(50)1000(100)2000(500)5000(1000)-20000(5000)30000(10000)50000, 100000, 500000, 1000000; 6D].

1916 (p. 97–107).—**24**. A. T. Doodson, S_n, C_n, S_n', C_n', $S_n^2 + C_n^2$, $S_n'^2 + C_n'^2$ and their logarithms for x = [1.1(0.1)1.9; 7S] and for 11 different values of n. Compare nos. **18**, **30**.

1916 (p. 108–122).—**25**. H. G. Savidge, (1) $Xb(x) = \text{ber}^2 x + \text{bei}^2 x$; (2) $Vb(x) = \text{ber}'^2 x + \text{bei}'^2 x$; (3) $Zb(x) = \text{ber}\, x\, \text{ber}'\, x + \text{bei}\, x\, \text{bei}'\, x$; (4) $Wbx = \text{ber}\, x\, \text{bei}'\, x - \text{bei}\, x\, \text{ber}'\, x$; (5)–(8) $Xk(x) = \text{ker}^2 x + \text{kei}^2 x$, $Vk(x)$, $-Zk(x)$, $-Wk(x)$; (9) $Vr(x) = \text{ber}'\, x\, \text{ker}'\, x + \text{bei}'\, x\, \text{kei}'\, x$. (10) $Vu(x) = \text{ker}'\, x\, \text{bei}'\, x - \text{ber}'\, x\, \text{kei}'\, x$; (11) (2)/(1); (12) (3)/(1); (13) (3)/(2); (14) (4)/(2). All of these functions are for the range x = [0.0(0.2)10.0; 5S–7S]. See no. **13**.

1916 (p. 123–126).—**26**. G. N. Watson, (a) $10 + \ln \Gamma(1 + x)$, for x = [.005(0.005)1.000; 10D]; (b) $10 + \int_0^x \log \Gamma(1 + t)dt$, for x = [0.1(0.1)-1.00; 10D]; (c) $d \ln \Gamma(x)/dx$, for x = [1(1)101; 13D] and x = [$1\frac{1}{2}$(1)$100\frac{1}{2}$; 13D].

1919 (p. 43–77).—**27**. R. L. Hippisley, Tables I–III of the lemniscate function, $K = K'$, and $K = 2K'$, $K = 4K'$, for r = [0(1)45; 10D]. "Some fault was found with the final decimals in the table of the lemniscate functions $K = K'$, to 7D, $\theta = 45'$, *1912*, p. 50. As this table has been used as the base of the calculation of the table for $K = (2, 3, 4, 5, 7 \cdots)K'$, by means of a transformation of the order 2, 3, 4, $\cdots$ as explained *1913*, p. 88, it was decided to make a recalculation to a higher degree of accuracy, going to 10 decimals." Compare no. **12**.

1919 (p. 78–79.—**28**. Table IV. $e^{-n\pi/16}$ for 60 integral values of n from $n = 1$ to $n = 242$, to 20D.

1919 (p. 80–81).—**29**. A. Lodge, Table V. Lemniscate seven-section, to 15D; Table VI. Lemniscate seventeen-section to 15D.

1922 (p. 263–270).—**30**. A. T. Doodson, Riccati-Bessel functions S_n, C_n, S_n', C_n', $S_n^2 + C_n^2$, $S_n'^2 + C_n'^2$, and their logarithms, for $x = [0.1(0.1)0.9$; 7S or 8S], and 8 different values of n. Compare no. **18**.

1922 (p. 271–272).—**31**. J. R. Airey, Zeros of Bessel functions $J_n(x)$ of high order, first 10 zeros, for $n = [0(1)10(5)20(10)50, 75, 100(100)500, 750, 1000$; 5D or 6D].

1923 (p. 287–289).—**32**. J. R. Airey, $\sin x$, and $\cos x$, $x = [0^r(1^r)100$; 15D]. Compare nos. **21**, **22**, **35**.

1923 (p. 290–293).—**33**. J. R. Airey, (a) $J_\nu(\nu)$, $J_{\nu-1}(\nu)$, $-Y_\nu(\nu)$, $-Y_{\nu-1}(\nu)$, $\nu = [0(1/4)10$; 6D]; (b) When ν is not an integer $J_\nu(x) = \frac{1}{\pi}\int_0^\pi \cos(\nu\theta - x\sin\theta)d\theta - \sin\frac{\nu\pi}{\pi}\int_0^\infty e^{-x\sinh\theta}e^{-\nu\theta}d\theta$, as shown by Schläfli. Let $F_\nu(\nu)$ denote the last term. $F_\nu(\nu)$, $F_{\nu-1}(\nu)$, $F_{-\nu}(\nu)$, $F_{-\nu+1}(\nu)$, for $\nu = [0(1/4)10$; 6D]; (c) Lommel-Weber Ω function, defined by $\Omega_\nu(x) = \frac{1}{\pi}\int_0^\pi \sin(x\sin\phi - \nu\phi)d\phi$, is tabulated for $\nu = [0(1/4)10; 6D]$. This definition of $\Omega_\nu(x)$ is that of Airey's table no. **36**, of Nielsen (1904), and of Jahnke and Emde (1938). Airey here erred in defining $\Omega_\nu(x)$ as the negative of this value.

1923 (p. 293–298).—**34**. A. E. Kennelly, Bessel functions of negative semi-imaginary argument, $z\sqrt{-i}$, $J_0(z\backslash 45°)$, $J_1\backslash 45°)$, for $z = [0(0.1)10$; 5D or 6D]. Also their application to determination of the alternating skin-effect impedance ratio, Z'/R, of straight uniform round copper wires, remote from other active conductors, according to the formula $Z'/R = [(z\backslash 45°)/2] \times J_0(z\backslash 45°)/J_1(z\backslash 45°)$, for $z = [0.0(0.1)10.0$; 6D].

1924 (p. 275–278).—**35**. J. R. Airey, Sin x, and cos x, for $x = [10.0(0.1)$-$20.0(0.5)50.0$; 15D]. Reprinted in B.A.A.S., *Mathematical Tables*, v. 1, London, 1931, as part of p. 4–7. Compare nos. **32**, **47**.

1924 (p. 279–287).—**36**. J. R. Airey, Lommel-Weber functions of zero and unit orders, $\Omega_0(x)$ and $\Omega_1(x)$ for $x = [0.00(0.02)16.00$; 6D]. Compare nos. **33**, **39**.

1924 (p. 287–295).—**37**. J. R. Airey, Bessel-Clifford functions of zero and unit orders, $C_0(x)$ and $C_1(x)$ for $x = [0.00(0.02)20.00$; 6D].

1925 (p. 221–233).—**38**. J. R. Airey, Bessel functions of half odd integral order, $J_\nu(x)$, to 12D; for (according to A. N. Lowan)

x	ν	x	ν	x	ν	x	ν
1	$\frac{-1(2)23}{2}$	6	$\frac{-15(2)47}{2}$	11	$\frac{-27(2)63}{2}$	16	$\frac{-37(2)79}{2}$
2	$\frac{-5(2)29}{2}$	7	$\frac{-17(2)51}{2}$	12	$\frac{-29(2)67}{2}$	17	$\frac{-39(2)81}{2}$
3	$\frac{-7(2)35}{2}$	8	$\frac{-19(2)53}{2}$	13	$\frac{-31(2)69}{2}$	18	$\frac{-41(2)85}{2}$
4	$\frac{-9(2)39}{2}$	9	$\frac{-21(2)57}{2}$	14	$\frac{-33(2)73}{2}$	19	$\frac{-43(2)87}{2}$
5	$\frac{-11(2)43}{2}$	10	$\frac{-23(2)61}{2}$	15	$\frac{-35(2)75}{2}$	20	$\frac{-45(2)89}{2}$

1925 (p. 234–253).—**39**. J. R. Airey, $J_{1/2}(x)$, $J_{-1/2}(x)$, $\Omega_{1/2}(x)$, $\Omega_{-1/2}(x)$, for $x = [0(0.02)20.00; 6D]$. Compare no. **36**.

1926 (p. 273–275).—**40**. J. R. Airey, Fresnel integrals, $S(x)$ and $C(x)$, for $x = [0.0(0.1)20.0; 6D]$.

1926 (p. 276–294).—**41**. J. R. Airey, Confluent hypergeometric function $M(\alpha\cdot\gamma\cdot x)$, for 31 values of $x = 0.0(0.1)1.0(0.2)3.0(0.5)8.0$, $\gamma = \pm 1/2$, $\pm 3/2$, $\alpha = [-4(1/2)4; 6D]$. Compare no. **43**.

1926 (p. 295–296).—**42**. J. R. Airey, $\sinh x$, $\cosh x$, for $x = [0.1(0.1)10.0; 15D]$. Reprinted in B.A.A.S., *Mathematical Tables*, v. 1, London, 1931, p. 30. Compare **47**(c), **52**.

1927 (p. 220–244).—**43**. J. R. Airey, Confluent hypergeometric function $M(\alpha\cdot\gamma\cdot x)$, $\gamma = 1, 2, 3, 4$, $\alpha = -4(0.5)4$, $x = [0.00(0.02)0.10(0.05)1.0(0.1)2.0(0.2)3.0(0.5)8.0; 6D \text{ or } 6S]$; and further values of the function for $\gamma = \pm 1/2, \pm 3/2$. Compare no. **41**, **55**.

1927 (p. 245–251).—**44**. J. R. Airey, Exponential, sine, and cosine integrals: (a) $Ei(\pm x)$, $x = [5.0(0.1)15.0; 5D \text{ to } 14D]$; (b) $\mathrm{Si}(x)$ and $\mathrm{Ci}(x)$, for $x = [5.0(0.1)20.0; 10D]$. Compare no. **48**.

1927 (p. 252–253).—**45**. J. R. Airey, Zeros of Bessel functions $J_\nu(x)$ of small fractional order, the first zero ρ_1, to 6S, for $\nu = -1.00(0.01)1.00$.

1927 (p. 253–254).—**46**. J. R. Airey, First 10 zeros, to 5D, of ber x, bei x, ker x, kei x, ber′ x, bei′ x, ker′ x, kei′ x.

1928 (p. 305–316).—**47**. J. R. Airey, (a) $\sin x$ and $\cos x$, for $x = [20.0(0.2)40.0; 15D]$; compare **32**; (b) $e^{\pi x}$, for $x = [-4.0(0.5)4.0; 20D]$, and (c) $\sinh \pi x$, $\cosh \pi x$, for $x = [0.01(0.01)4.00; 15D]$. In B.A.A.S., *Mathematical Tables*, v. 1, Cambridge, 1931, p. 5–6, 26–29, (a) and (c) were reprinted.

1928 (p. 316–323).—**48**. J. R. Airey, (a) $\mathrm{Si}(x)$ and $\mathrm{Ci}(x)$, for $x = [20.0(0.2)40.0; 10D]$; compare no. **44**; (b) Derivatives of Bessel functions $\frac{\partial}{\partial\nu} J_\nu(x)$, $x = [0.0(0.1)20.0; 6D]$, $\nu = \pm 1/2, \pm 3/2$. Such derivatives are closely related to sine and cosine integrals. For nos. **44**, and **48**(a) see B.A.A.S., *Mathematical Tables*, v. 1, Cambridge, 1931, p. xv, etc.

1928 (p. 324–340).—**49**. J. R. Airey, The probability integral $I_0 = \int_x^\infty e^{-(1/2)t^2}\,dt$ and its integrals. Let $I_n = \int_x^\infty I_{n-1}(x)dx$. The tables are for $I_n(x)$, for $n = 0, 1, \cdots 20$, $x = [0.0(0.1)6.6; 10D]$; and for $I_n(-x)$, $n = 0, 1, 2, \cdots 21$, for $x = [0.0(0.1)10.0; 10D]$.

1929 (p. 251–262).—**50**. A. Lodge, Harmonic series $\phi(x)$, for $x = [0.0(0.1)60.3; 16D]$ and $x = [50.00(0.01)51.00; 10D]$. $\phi(x) = \gamma + \frac{d}{dx}\ln\Gamma(1+x)$, where γ is Euler's constant. *This is the last table published in the B.A.A.S. Reports.*

In its report of 1929 the Committee stated that the "following Tables from other sources are intended to supplement those which have appeared in the Committee's reports" to the B.A.A.S.:

51. J. W. L. Glaisher, "Tables of the numerical values of the sine-integral, the cosine-integral, and exponential-integral," R. So. London, *Trans.*, v. 160, 1870, p. 367–388. Tables I–IX: $\mathrm{Si}(x)$, $\mathrm{Ci}(x)$, $\mathrm{Ei}(\pm x)$, for $x = [0(0.01)1; 18D]$, $[1(0.1)5(1)15; 11D]$, with differences to the third order up to $x = 5$; Table X: $\mathrm{Si}(x)$, $\mathrm{Ci}(x)$, for $x = [20(5)100(10)200(100)1000$-

(1000)11000; 7D], $x = [10^5, 10^6, 10^7, 10^8, \infty$; 7D]. Tables XI, XII: maxima and minima of Si(x) and Ci(x), to 7D. The "Table of Constants" (p. 370) gives the values of $x\Gamma(x+1)$ and its ln, for x = [2(1)71; 20S, and 10D respectively].

52. D. F. E. Meissel, Tafel der Bessel'schen Functionen I_k^0 und I_k^1 von $k = 0$ bis $k = 15.5$ berechnet," Preuss. Akad. Wissen., *Abh.* 1888, Berlin, 1889, 23 p. I_k^0, I_k^1 are simply $J_0(x)$, $J_1(x)$. The first 10 zeros of $J_0(x)$ are given on page 3 and the values of $J_0(x)$, $J_1(x)$ are given (p. 4–23), for x = [0.00(0.01)15.50; 15D]. Reprinted as Table I, in A. Gray and G. B. Mathews, *A Treatise on Bessel Functions*, second ed. by A. Gray and T. M. Macrobert, London, 1922, p. 267–286. A correction of $J_0(1.71)$, suggested by Meissel, is here made, but there were three other errors in Meissel

$J_0(0.62)$,	*for*	0.90518,	*read*	0.90618
$J_0(1.89)$,	*for*	0.28663,	*read*	0.28763
$J_1(7.87)$,	*for*	0.21401,	*read*	0.21407

The first and third of these, as well as $J_0(3.07)$, which is correct in Meissel, are incorrect in Gray and Mathews 2. The second is incorrect in G. and M.1. Table II (p. 286–299), of Gray and Mathews, is from an unpublished ms. of Meissel of $J_n(x)$ for $x = 0(1)24$, n = [0(1)60; 16D]. In this table Airey (no. **42**, p. 297) noted the following errors: $J_4(5)$ *for* 26304, *read* 23604; $J_{23}(6)$ *for* 02496, *read* 02495; $J_{30}(14)$ *for* 538, *read* 534; $J_{31}(16)$ *for* 49322, *read* 94322. Another error in G. and M.1 is $J_0(5.90)$, *for* 0.11203, *read* 0.12203. Table IV of Gray and Mathews is taken from D. F. E. Meissel, "Ueber die Bessel'schen Functionen J_k^0 und J_k^1." *Progr.*, Kiel, 1890, and contains the first 50 roots of $J_1(x) = 0$, with the corresponding maximum or minimum values of $J_0(x)$.

53. R. W. Willson and B. O. Peirce, "Table of the first forty roots of the Bessel equation $J_0(x) = 0$ with the corresponding values of $J_1(x)$," Amer. Math. So., *Bull.*, v. 3, 1896, p. 153–155. The value of $J_1(x)$ for the 35th zero is incorrect; *for* 0.07635913, *read* 0.07636383. Reprinted with the error in Gray and Mathews.

54. H. E. Wrinch and D. M. Wrinch, "Tables of Bessel functions," *Phil. Mag.*, s. 6, v. 47, 1924, p. 62–65, $I_n(x)$, for large integral values of x from 16 to 37 with tables of $J_2(x)/I_2(x)$, and $J_2(x)/I_3(x)$, for x from 1 to 15.

55. H. E. Wrinch and D. M. Wrinch, "Roots of hypergeometric functions with a numerator and four denominators," *Phil. Mag.*, s. 7, v. 1, 1926, p. 273–276. First 9 roots for various values of the five parameters. Compare no. **41**.

56. W. S. Aldis, "Tables for the solution of the equation $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \left(1 + \frac{y^2}{x^2}\right)y = 0$," R. So. London, *Proc.*, v. 64, 1899, p. 203–223. $I_0(x)$, $I_1(x)$, for x = [0.1(0.1)6.0(1)11; 18D to 21D]; $K_0(x)$, $K_1(x)$, for x = [0.1(0.1)120; 9D to 21D].

57. A. Lodge, Zonal harmonics, $P_n(\cos\theta)$, $\theta = 0°(5°)90°$, n = [1(1)20; 7D], R. So. London, *Trans.*, v. 203 A, 1904, p. 100–101; "last figure approximate." Appendix to a paper, "On the acoustic shadow of a sphere," by Rayleigh. Compare RMT **92**.

58. J. C. ADAMS, "Useful formulae, connecting Legendre's coefficients, which are employed in the theory of terrestrial magnetism," *Scientific Papers* of Adams, v. 2, Cambridge, 1900, p. 243–296. On p. 268–281, Gaussian functions $G_n^m(\mu)$, $n = 0(1)10$, $m = 0(1)10$, $\mu = [0.00(0.05)1; 10D]$.

Authors

R. C. A.

RECENT MATHEMATICAL TABLES

98[S].—PROJECT FOR COMPUTATION OF MATHEMATICAL TABLES (A. N. Lowan, technical director), *Miscellaneous Physical Tables. Planck's Radiation Functions, and Electronic Functions.* Prepared by the Federal Works Agency, Works Projects Administration for the City of New York, conducted under the sponsorship of the National Bureau of Standards. New York, 1941, vii, 58 p. 20.2 × 26.3 cm. Reproduced by a photo offset process. Sold by the U. S. Bureau of Standards, Washington, D. C. The work was not distributed until 1943. $1.50; foreign price $1.75.

The tables of Radiation Functions are reprinted from Optical So. Amer., *Jn.*, Feb. 1940. A body of alsolute temperature T emits electromagnetic waves of all possible wave lengths λ. The radiated energy is however very unequally distributed among the waves of different length. The radiation and photon functions are defined as

$$R_\lambda = c_1\lambda^{-5}(e^{c_2/\lambda T} - 1)^{-1};\ R_{0-\lambda} = \int_0^\lambda R_\lambda d\lambda;\ N_\lambda = 2\pi c\lambda^{-4}(e^{c_2/\lambda T} - 1)^{-1};\ N_{0-\lambda} = \int_0^\lambda N_\lambda d\lambda.$$

In a region of wave lengths ranging from zero to λ the integrals $R_{0-\lambda}$ and $N_{0-\lambda}$ express, respectively, the rates of emission of energy and photons per units of area and time. The letters c, c_1, c_2 have special numerical values. It is chiefly in the values of these constants that there is a difference between the various tables previously published and listed on p. 12–13; c represents the velocity of light (cm./sec.). The values of c_1 and c_2 which were adopted after consultation with L. J. Briggs and H. T. Wensel of the National Bureau of Standards use numbers 3.732 and 1.436 while the recent tables of P. Moon [1] use 3.697 and 1.432 respectively. On p. 4 of the introduction a way of correcting for a change in c_1 and c_2 is explained. Remarks are made also on the number of significant figures in the tabulated entries and on the method of interpolation for λ. In making the computations results obtained by means of series were checked by evaluating the integrals by a method of numerical integration. In Table I we are given values of $R_{0-\lambda}/R_{0-\infty}$, $R_\lambda/R_{\lambda\max}$, $N_{0-\lambda}/N_{0-\infty}$, $N_\lambda/N_{\lambda\max}$, for $\lambda T = 0.050(0.001)\cdot 1(0.005).4(0.01).6(.2)1(0.05)2$. In Table II are values of $R_{0-\lambda}$, R_λ, $N_{0-\lambda}$, N_λ, for λ (in microns) $= .5(0.01)1(0.05)4(0.1)6(0.2)10(0.5)20$. In Table III are given values of N_λ, to 4D, for T = 1000°K, 1500°K, 2000°K, 2500°K, 3000°K, 3500°K, 6000°K, and $\lambda = 0.25(0.05)1.6(0.2)3(1)10$. The last table is to be used for changes in c_1 and c_2, from 4D to 5D are given for $\lambda T = 0.025(.005).13(.01).2(.05).6(.1)1(.5)2$.

For an electron of velocity v, charge e in electromagnetic units and mass m_0, let $x = v/c$, where c is the velocity of light, let $G = (1 - x^2)^{-1/2}$. The need of tables of the function G has been felt for a long time. In his work on the distribution of electricity on a thin circular disc, George Green [2] gave a 3-place table for $x = 0(.2).8(.1)1$. A more extensive 3-place table for $x = 0(.01).4$ was prepared by M. Hamy [3] for use in the theory of perturbations. When the function occurred in the expressions for mass and energy in electromagnetic theory more extensive tables were needed. G. Fournier [4] gave from 5 to 6S for $x = 0(.01).9(.02)$-$.99(.001).995(.0005)1$ while E. N. Da C. Andrade [5] gave from 3 to 4S for $x = 0(.005)$-$.01(.01).05(.05).8(.01).99, .995, .998$.

The present tables were computed at the suggestion of A. E. Ruark. From 7 to 10D are given for the range $x = .005(.005)1(.001).9(.0005).96(.0002).99(.0001).995(.00005)$-$.998(.00002).999(.00001).99999(.000001).999999(.0000001).9999999(.00000001).99999999$ $(.000000001).999999996,.999999997(.0000000005).999999998(.0000000001).9999999990$. Corresponding to these values of x are given not only the values of G, but also of Gx, and $V = 10^{-8}e^{-1}T$ [where T the kinetic energy $= m_0c^2(G - 1)$, m_0 being the mass; Einstein's formula], and $H_\rho = (m_0/e)cxG$.

H. B.

[1] P. Moon, "A table of Planck's function from 3500 to 8000K," *Jn. Math. Phys.*, Mass. Inst. Tech., v. 16, 1938, p. 133–157.

[2] G. Green, "Mathematical investigations concerning the laws of the equilibrium of fluids analogous to the electric fluid with other similar researches," Cambridge Phil. So., *Trans.*, v. 5, 1833, p. 1–63; table on p. 62.

[3] M. Hamy, "Sur le développement approché de la fonction perturbatrice dans le cas des inégalités d'ordre élevé," *Jn. de Math.*, s. 4, v. 10, 1894, p. 391–472, s. 5, v. 2, 1896, p. 381–439; table on p. 466.

[4] G. Fournier, "Tables relatives à l'électron," *Jn. de Physique*, s. 6, v. 6, 1925, p. 23–32.

[5] E. N. da C. Andrade, *The structure of the atom*, London, Bell, 1924, p. 300; third ed., 1927, p. 720.

99[J, K, L].—CATHERINE M. THOMPSON, "Tables of percentage points of the incomplete beta-function," *Biometrika*, v. 32, 1941, p. 168–181. 19.3 × 27.3 cm.

The incomplete beta-function is here defined as

$$I_x(p, q) = \frac{\Gamma(p + q)}{\Gamma(p)\Gamma(q)} \int_0^x x^{p-1}(1 - x)^{q-1}dx.$$

The percentage points of this function are the values of x which satisfy the integral equation

$$I_x(p, q) = P$$

for given values of P, p, and q.

A table of x is given for each of seven values of P: 0.50, 0.25, 0.10, 0.05, 0.025, 0.01, and 0.005. For each P, $\nu_1 = 2q = 1(1)\ 10, 12, 15, 20, 24, 30, 40, 60, 120, \infty$; $\nu_2 = 2p = 1(1)$ $30, 40, 60, 120, \infty$; x to 5S (in most parts of the table this is equivalent to 5D).

In a prefatory note (p. 151–153), E. S. Pearson points out some uses of the Table in statistical problems. In particular, if S_1, S_2 are two independent sums of squares, distributed as $\chi^2\sigma^2$ with ν_1 and ν_2 degrees of freedom respectively, the table gives the percentage points of

$$x = S_2/(S_1 + S_2).$$

The multiple correlation coefficient R^2 follows the beta-function distribution when the population is normal with zero correlation. The beta-function distribution also serves frequently as an approximation to the distribution of a random variable x which is constrained to lie between 0 and 1.

Partial sums of a binomial expansion are expressible in terms of an incomplete beta-function. From a result due to Laplace, *Théorie Analytique des Probabilités*, 2nd ed., Paris, 1814, p. 151, the sum of the first s terms of the binomial $(q' + p')^n$, where $q' + p' = 1$,

is found to be

$$I_q'(n + 1 - s, s).$$

Thus the present tables, if read at $\nu_1 = 2s$, $\nu_2 = 2(n + 1 - s)$, provide the value of q' for which the sum has the given value P. This result may be used to construct fiducial limits for the probability p' of occurrence of an event which has been observed to happen $(s - 1)$ times in $(n - 1)$ trials.

The first table of this type was constructed by R. A. Fisher, *Statistical Methods for Research Workers*, Edinburgh, Oliver and Boyd, 1925. For $P = 0.05$, $n_1, n_2 = 1(1)6, 8, 12, 24, \infty$, Fisher's table gave the values of z to 4D, where $z = (1/2) \ln F$, $F = [p(1 - x)]/qx$. In the 2nd ed. (1928) of Fisher's work a table for $P = 0.01$ is also given; and in the 8th ed. (1941), a further table for $P = 0.001$, is credited to C. G. Colcord and L. S. Deming (1936). In Fisher's notation n_1, n_2 replace ν_1, ν_2 respectively. These tables were subsequently extended to $P = 0.20$ (H. W. Norton, Iowa State College Master's thesis, 1937). These tables, for $P = 0.20, 0.05, 0.01, 0.001$, $n_1 = 1(1)6, 8, 12, 24, \infty$; $n_2 = 1(1)30, 40, 60, 120, \infty$; z to 4D, appear in: R. A. Fisher and F. Yates, *Statistical Tables for Biological, Agricultural and Medical Research*, Edinburgh, Oliver and Boyd, 1938.

From Fisher's table, P. C. Mahalanobis, *Indian Jn. Agric. Sci.*, v. 2, 1932, p. 678–693, constructed tables of $z' = (1/2) \log F$; $y = F^{1/2}$. In these tables, $P = 0.05, 0.01$; $n_1 = 1(1)6, 8, 12, 24, \infty$; $n_2 = 1(1)30, 60, \infty$; z' to 4D; F to 3D (throughout most of the table); y to 3D (throughout most of the table). Where no interpolation is required, the table of F (in statistical terminology the *variance ratio*) is usually the most convenient for tests of significance associated with the analysis of variance. A further table of F, for $P = 0.20, 0.05, 0.01, 0.001$; $n_1 = 1(1)6, 8, 12, 24, \infty$; $n_2 = 1(1)30, 40, 60, 120, \infty$; F to 2D in most parts of the table, is given by Fisher and Yates, *loc. cit.* Miss Thompson's tables do not include the values $P = 0.20, 0.001$; in this respect Fisher and Yates's tables and the present tables supplement each other.

The article by Miss Thompson also contains a description of the methods of computation by L. J. Comrie and H. O. Hartley (p. 154–161), who advised on this phase of the work, and an account (p. 161–167) by H. O. Hartley of suitable methods of interpolation for users of the tables.

W. G. Cochran

100[H, J, K].—L. J. Comrie and H. O. Hartley, "Table of Lagrangian coefficients for harmonic interpolation in certain tables of percentage points," *Biometrika*, v. 32, 1941, p. 183–186. 19.3 × 27.3 cm.

In this table the Lagrange coefficients have been calculated by the standard formulas for interpolation by a polynomial of sixth degree. However, in determining Lagrange coefficients for harmonic interpolation, the coefficients are those for polynomials of the sixth degree in the reciprocal of the parameters used as the argument. The table of coefficients is so constructed that the reciprocals of the numbers in the following progression (or any submultiple progression) may be used as arguments of tabular values: 10, 12, 15, 20, 24, 30, 40, 60, 120, ∞. The Lagrange coefficients are given to five places of decimals and for the following values of the interpolate: 16(1)19 with 10, 12, 15, 20, 24, 30, 40 as the arguments of tabular values 21(1)23 with 12, 15, 20, 24, 30, 40, 60 as the arguments of tabular values, 25(1)29 with 15, 20, 24, 30, 40, 60, 120 as the arguments of tabular values 31(1)119 (excluding 40 and 60) with 20, 24, 30, 40, 60, 120, ∞ as the arguments of tabular values.

For smaller values of the interpolate (i.e., 11 to 19) ordinary Lagrange coefficients (to five places of decimals) are given for sixth degree polynomial interpolation since the parameter itself rather than its reciprocal is preferable as the argument in this range.

The table of Lagrange coefficients for harmonic interpolation are very useful for finding percentage points of the incomplete beta-function and Fisher's z distribution for new combinations of values of the arguments.

Illustrative examples are given, showing how to apply the tables to these two functions.

S. S. W.

101[J, L].—C. M. Thompson, "Table of percentage of the χ^2 distribution," *Biometrika*, v. 32, 1941, p. 187–191. 19.3 × 27.3 cm.

The probability distribution of χ^2 with ν degrees of freedom is

$$f(\chi^2) = \frac{(1/2)^{\nu/2}}{\Gamma(\frac{1}{2}\nu)}(\chi^2)^{\frac{1}{2}\nu-1}e^{-\frac{1}{2}\chi^2}$$

The percentage points of this distribution are the values of χ^2 which satisfy the equation

$$\int_{\chi^2}^{\infty} f(\chi^2)d(\chi^2) = P$$

for given values of P, ν. In the present table P = 0.995, 0.99, 0.975, 0.95, 0.9, 0.75, 0.5, 0.25, 0.1, 0.05, 0.025, 0.01, 0.005; $\nu = 1(1)30(10)100$; χ^2 to 6S. The editor states that for $\nu > 50$ the sixth figure may be in error by one or two units. Interpolation formulae are suggested both for the body of the table and for values of ν outside the range of the table.

Apart from the normal frequency distribution, that of χ^2 is perhaps the most frequently encountered in modern statistical theory. Under certain assumptions about the nature of the population, the distribution of an estimated variance s^2, derived from ν degrees of freedom, is related to that of χ^2 by the equation $\nu s^2 = \chi^2\sigma^2$, where σ^2 is the population variance of which s^2 is an estimate. The percentage points of the χ^2 distribution are accordingly used to calculate fiducial limits for σ^2 from a given estimate s^2. The χ^2 distribution serves also as an approximation to many distributions whose exact forms are not yet known or have not been tabulated. In this connection the most important instances are the uses of χ^2 in K. Pearson's test of "goodness of fit" of an observed to a theoretical frequency distribution and in the tests of homogeneity of a binomial distribution, of a Poisson distribution and of a contingency table.

The χ^2 distribution is closely related to the partial sum of a Poisson frequency distribution. The sum of the first s terms of the latter, with mean m, satisfies the relation

$$e^{-m}\left\{1 + m + \frac{m^2}{2!} + \cdots + \frac{m^{s-1}}{(s-1)!}\right\} = \int_{2m}^{\infty} f(\chi^2)d(\chi^2)$$

where the number of degrees of freedom in χ^2 is $\nu = 2s$. It follows that from the percentage points of the χ^2 distribution the value of m can be found for which the sum of a given number of terms of the Poisson series has a given value.

The first table of the percentage points of χ^2 was published by R. A. Fisher, *Statistical Methods for Research Workers*, Edinburgh, 1925. A recent edition of this table, in R. A. Fisher and F. Yates, *Statistical Tables for Agricultural, Biological and Medical Research*, Edinburgh, 1938, gives χ^2 to 3D (except for some small values of n, where 3S are given) for P = 0.99, 0.98, 0.95, 0.90, 0.80, 0.70, 0.50, 0.30, 0.20, 0.10, 0.05, 0.02, 0.01, 0.001; $n = 1(1)30$. Except for $P = 0.001$, the same table is to be found in the eighth edition of Fisher's *Statistical Methods*, 1941. Fisher uses n in place of ν. It should be noted that Fisher's table and Miss Thompson's table cover somewhat different values of P.

W. G. Cochran

102[K].—M. Merrington and C. M. Thompson, "Tables of percentage points of the inverted beta (F) distribution," *Biometrika*, v. 33, 1943, 16 p. 19.3 × 27.3 cm. Compare RMT **99**.

By making the transformation

$$x = \frac{1}{1+u} \tag{1}$$

on the incomplete beta distribution

$$f(x) = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)}x^{p-1}(1-x)^{q-1}, \qquad (0 < x < 1) \tag{2}$$

we obtain the so-called inverted beta distribution

$$(3)\qquad g(u) = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} u^{q-1}(1+u)^{-p-q}, \qquad (0 < u < \infty).$$

If χ_1^2 and χ_2^2 are independently distributed according to χ^2-laws with ν_1 and ν_2 degrees of freedom respectively, the χ^2-law with ν degrees of freedom respectively being defined by

$$(4)\qquad \frac{1}{2}\frac{(\frac{1}{2}\chi^2)^{(\nu/2)-1}}{\Gamma(\frac{1}{2}\nu)} e^{-(1/2)\chi^2} d(\chi^2),$$

then $\chi_1^2/\chi_2^2 = u$ is distributed according to $g(u)$ with $q = \frac{1}{2}\nu_1$, $p = \frac{1}{2}\nu_2$. But $\chi_1^2/\chi_2^2 = (\nu_1/\nu_2)F$, where F is the Snedecor ratio which is the most convenient form for the analysis of variance test criterion. Hence the distribution law of F, say $h(F)dF$, may be found by making the transformation $u = (\nu_1/\nu_2)F$ in (3) with $q = (1/2)\nu_1$, and $p = (1/2)\nu_2$. Let

$$I_F(\nu_1, \nu_2) = \int_F^\infty h(F)dF.$$

Values of F are tabulated to five significant figures, for $I_F(\nu_1, \nu_2) = 0.50, 0.25, 0.10, 0.05, 0.025, 0.01, 0.005$ and for $\nu_1 = 1(1)10, 12, 15, 24, 30, 40, 60, 120$, $\nu_2 = 1(1)30, 40, 60, 120, \infty$. Harmonic interpolation is therefore possible for both ν_1 and ν_2.

S. S. W.

103[Q, U].—J. Y. Dreisonstok, *Navigation Tables for Mariners and Aviators*, sixth ed., 1942, iv, 109 p. 14.9 × 23.2 cm. (H. O. no. 208.) For sale by the Hydrographic Office and by the Superintendent of Documents, Washington, D. C. $1.20; foreign price, postage extra.

These tables were originally designed by Dreisonstok, a Lieut. Commander, U.S.N.; the sixth edition was modified and revised by E. B. Collins, a Senior Scientist, U. S. Hydrographic Office. They have been popular and widely used; for a time they were standard equipment in the British Royal Air Force. Except for a number of large errors, their accuracy is entirely adequate for surface navigation (altitudes are accurate to about four-tenths of a minute of arc when the tables are used without interpolation, one-tenth with) and more than adequate for aerial navigation. Compare MTE 11.

To simplify the discussion of these and other navigation tables, the following paragraph on notation is presented. The astronomical triangle is the triangle on the infinite celestial sphere with vertices at the observer's zenith, Z, the celestial object under consideration, S, and the visible celestial pole, P. In the usual notation of navigators, $\angle SPZ = t =$ local hour angle; $\angle PZS = Z =$ azimuth; side $PZ = 90° - L$ where L is the observer's latitude; side $PS = 90° - d$ where d is the declination of the celestial object; side $ZS = 90° - h$ where h is the altitude of the celestial object.

The fundamental problem in celestial navigation as commonly practised today is this: Given t, d, and L; to find h and Z. The computed altitude, h, is compared with an observed altitude, properly corrected. Using the computed azimuth, Z, and the difference between the observed and computed altitudes, a "line of position" is drawn; this is used as the locus of possible positions of the observer.

Dreisonstok drops a perpendicular from Z on the great circle through P and S, meeting it at E. This perpendicular divides the astronomical triangle into two right triangles, the polar triangle containing P and the stellar triangle containing S.

Tables I and IA are double-entry tables in which are tabulated four quantities associated with the polar triangle, b, A, C, and Z', with arguments L and t, each given for integral degrees, t 0° to 360°. $b =$ side PE (degrees, minutes and tenths); $A = 10^5 \log \sec a$ (to nearest unit); $C = 10^5 \log \csc a$ (to nearest unit) where $a =$ side ZE; and $Z' = PZE$ (degrees and tenths). Table I contains values for latitudes (L) 0° to 65° North or South; Table IA is for latitudes 66° to 90°. It may be noted that Table IA was first computed by E. B. Collins for the Byrd Antarctic expedition in 1929; this table is included in the third and sixth editions, not in the others.

Table II is essentially a table of logarithms of cosecants and cotangents for every minute of arc, 0° to 180°.

$B = 10^5 \log \csc (b + d)$; $D = 10^5 \log \cot (b + d)$, each given to the nearest integer.

h and Z are obtained by the formulas:

$$10^5 \log \csc h = A + B, \qquad 10^5 \log \tan Z'' = C + D, \qquad Z = Z' + Z''.$$

Dreisonstok's method is known as an "assumed position" (or A.P.) method since the altitude and azimuth are computed for a position with integral values (in degrees) of t and L. This position may in the limit be as much as 42 nautical miles from the dead-reckoning position. In most cases in practise, this is satisfactory. In general, this method, like other A.P. methods, is not very satisfactory when the observed altitude is near 90°; the azimuth is poorly determined in these cases.

There are two ways in which the form of the tables could be improved. Tables I and IA might be united in a single table with the same form throughout. Secondly, all the values corresponding to a single latitude might be brought together on a single page; at present values corresponding to a single hour-angle are brought together on two pages which are widely separated in the volume. A reduction in page-turning would result from these changes since hour-angle varies more rapidly and more widely than does latitude. In a letter, the Chief Hydrographer has stated that the values in Table I and IA have been re-computed using seven-place logarithms. It is to be hoped that when the volume is again set up in type, the tabular values may be correct to the last place given and that the two suggested changes in form may be made.

The tables may be used for the determination of great circle course and distance and for star identification. See further RMT **106**.

Brown University C. H. SMILEY

104[Q, U].—A. A. AGETON, *Dead Reckoning Altitude and Azimuth Table*, third ed., Washington, D. C., 1940, v, 49 p. 14.5 × 23.4 cm. (H.O. no. 211.) For sale by the Hydrographic Office and by the Superintendent of Documents, Washington, D. C. $.90; foreign price, postage extra.

This is a table neither of solutions nor of partial solutions of the astronomical triangle; instead it is essentially a table of logarithms of secants and cosecants with which the altitude and azimuth of a celestial body may rapidly and accurately be computed from the local hour angle, declination and latitude. The chief advantages of the table lie in the elegant simplicity of the working rules devised by Ageton and in the fact that the computations are made for the dead reckoning position. Because of the latter fact, Ageton's method is known as a D.R.P. method and, for a celestial object near the zenith, it is better than an A.P. method.

In the notation of RMT **103**, Ageton drops a perpendicular from S on the great circle through P and Z, meeting it at X. This perpendicular divides the astronomical triangle into two right triangles, the zenith triangle and the polar triangle. In the polar triangle, the side PX is called $90° - K$ and the side SX is denoted by R. In the zenith triangle, $K \sim L$ is used to denote $|K - L|$.

$A(x)$ and $B(x)$ are defined as follows: $A(x) = 10^5 \log \csc x$; $B(x) = 10^5 \log \sec x$. These functions are given to the nearest integer for each half-minute of arc from 0° to 180°, except that when the value of a function is less than 240, one decimal is given. Because of the tabulation of the functions for every half-minute, interpolation is not required for ordinary navigation. By the introduction of the factor of 10^5, the complication of a decimal point is eliminated. The adoption of the quadrantal form for the table simplifies the matter of entering it for a function without making the volume bulky. The B's are printed in a heavy-face type, the A's in a light-face type, thereby reducing the possibility of error. As is true of all Hydrographic Office tables, the volume is well printed on good paper.

The basic formulas are:

$$A(t) + B(d) = A(R); \qquad A(d) - B(R) = A(K);$$
$$B(R) + B(K \sim L) = A(h); \qquad A(R) - B(h) = A(Z).$$

There are only three places in the computation where judgment is required, namely when K, h, and Z are being taken from the table. Ageton has prepared simple rules to cover these points and two of the rules are printed at the top of every double-page through the table.

It may be pointed out that Ageton's basic formulas and precepts will work equally well if $A(x)$ is defined as log sin x and $B(x)$ as log cos x. Thus an ordinary five-place logarithm table can be used when a copy of H. O. 211 is not available, or a seven-place table can be used when greater accuracy is desired.

L. J. Comrie has commented (*Hughes' Tables for Sea and Air Navigation*, London, 1938, p. xxxi) that Ageton's table "has been prepared from a six-figure table, with no attempt to ascertain whether a five in the sixth decimal should be rounded up or down; consequently the last printed figure is in error in one out of twenty entries, although admittedly these errors are of no real consequence."

As for indeterminacy, Ageton has printed on the page facing the title page, the warning that when the local hour angle is near 90°, an error of one or two (nautical) miles may occur in computing an altitude. In every method where the altitude is determined by a single trigonometric function, there will be a region of indeterminacy; the one occurring here is no more troublesome in practise than most of them.

This table may also be used for the computation of the initial course and distance along a great circle between two terrestrial points, for star identification, and for locating points on a great circle track. In astronomical work, it may be used to determine the orientation of the crescent moon with reference to the horizon, the distance between two celestial objects whose positions are known, and to transform one set of spherical coördinates into another, and for other similar problems. See further RMT **106**.

C. H. Smiley

105[Q, U].—*Tables of Computed Altitude and Azimuth, Latitudes, 0° to 9°, inclusive*, v. 1, Washington, D. C., 1941, 3, xi, 2–253 p. 22.6 × 29.1 cm. This is the first of eight uniform v. of H.O. 214, each v. devoted to 10° of latitude, v. 8 for 70°–79°. V. 2, 4, 5, 6, 7 were each published in 1940, and each contained 3, xi, 2–265 p.; v. 8, 1941, 3, xiii, 2–265 p. For sale by the Hydrographic Office and by the Superintendent of Documents, Washington, D. C. $2.25 per v.; foreign price, postage extra.

These tables present complete solutions of the astronomical triangle. Their principal advantages are the rapidity and simplicity of use and the accuracy of results; their disadvantages are their weight and bulk. Since each volume includes both cases, declination same name as latitude and contrary name, the eight cover all latitudes between 80°S and 80°N. Volume 8 has a special section of two pages devoted to polar navigation. The material for volume 9 has been prepared but, because of the limited use to which such a table for latitudes 80°–90° would be put, and because of the expense, it is not planned to publish it.

Volume 4 was the first one to be completed; it appeared in 1936. Copies of it were sent to many navigators and their criticisms and suggestions were invited. The other volumes were then prepared with the assistance of the Philadelphia group of the Works Progress Administration.

Within the section devoted to a particular degree of latitude, altitude (to 0′.1) and azimuth (to 0°.1) are tabulated for every degree of local hour angle, 0° to 180° or down to altitude 5°, and for selected values of the declination. The values of declination include every half-degree from 0° to 29° inclusive; these allow for the convenient determination of altitude and azimuth for the sun, moon and planets as well as stars of declination numerically less

than 29°.5. Beyond 29°, the following 37 declinations are listed: 30°, 32°, 34°, 34°.5, 35°.5, 36°, 37°, 38°.5,40°, 42°, 42°.5, 43°, 45°, 46°, 47°, 48°.5, 49°.5, 50°.5, 51°.5, 52°.5, 54°, 54°.5, 55°, 56°, 56°.5, 57°, 57°.5, 59°, 59°.5, 60°, 60°.5, 62°, 62°.5, 63°, 69°, 69°.5, 74°.5. These values were chosen for the convenient reduction of observations of a selected list of stars, although the list is not given in the volumes.

For each latitude, a star identification table of two pages is given, tabulating declination and hour angle in integral degrees with arguments altitude 4°(4°)88°, and azimuth 0°(4°)180°.

In addition to altitude and azimuth for each entry, there are given the changes in altitude per minute of arc of declination, and per minute of arc of hour angle. With the aid of a multiplication table on the inside of the back cover and the page facing it, one can easily interpolate the altitude for the precise declination and hour angle and for an integral latitude. Another table is given for correcting the altitude for small changes in latitude; thus one may determine the altitude of a body for the dead reckoning position. There are no facilities for determining the azimuth for this position and it is recommended that one not attempt to use the tables for the dead reckoning position when the altitude is greater than 80°.

If one assumes that the tabular altitude and each of the three corrections are in error by 0′.1 and all of the errors of the same sign, the maximum error of an altitude for a dead reckoning position would be 0′.4. Generally one would expect the error to be 0′.2 or less.

In the Description of the Tables (p. iv), it is stated that "the altitudes have been computed to an accuracy of one tenth of a minute of arc by seven-place logarithms."[1] The reviewer found six errors, each of two tenths of a minute of arc, in a very brief check of a small part of volume 4; the amount of material checked was too small to serve as a basis for an estimate of the number and magnitude of the errors in the entire set of tables. In response to a query, the Chief Hydrographer has written that the tables were checked, but he gave no information as to the method or the thoroughness of the check. See further RMT **106**.

C. H. SMILEY

[1] In computing the nine volumes of H. O. 214, the Hydrographic Office used 100 copies of Peters' *Siebenstellige Logarithmentafel der trigonometrischen Funktionen für jede Bogensekunde des Quadranten*, Leipzig, 1911. After the completion of the work copies of the tables were distributed to various Government departments.—EDITOR.

106[Q, U].—*Astronomical Navigation Tables, Latitudes* 65°–69°, *North and South, Volume P*, Washington, D. C., U. S. Government Printing Office, 1941, 233 p. 16.5 × 24.6 cm. This is the fourteenth and last v. of H.O.218, each one covering five degrees of latitude, v. A, 0°–4°, no volume lettered I or O. Not available for public distribution or sale.

These tables are "reproduced by photo-lithographic process for emergency use from British Air Publication 1618," of the same name. The reproduction was "undertaken to meet the emergency requirements of U. S. Naval Aviation and the U. S. Army Air Corps in cooperating with the British Air forces." The set is useful only between 70° South and 70° North latitude. Persons who have used these tables as well as other modern tables agree that they are the fastest tables available for navigational purposes at the present time. Altitudes obtained from them are said to be correct to the nearest minute of arc, an accuracy entirely adequate for aerial use for which they are intended, although not as close as many navigation tables now in common use.

In the front of each volume, about 89 pages are devoted to 22 bright navigation stars. Altitude and azimuth of each star are tabulated to the nearest minute and the nearest degree respectively with arguments latitude (the five degrees covered by the volume) and hour angle (0° to 180° or down to 10° altitude) given to integral degrees. For each entry, a value is given (called *t* but *not* the local hour angle) which serves as an argument in entering a table printed on the inside of the front cover, to correct tabulated altitudes for any year to 2000 A.D. inclusive. No correction is needed in most cases before 1944.

In the latter part of each volume are tables in which for an integral value of declination (in degrees, 0° to 28° inclusive) altitude and azimuth are tabulated to the nearest minute and the nearest degree respectively with arguments latitude (again for the five degrees covered by the volume) and local hour angle (0° to 180° or down to 10° altitude) given to integral degrees. For each entry in the tables, a value of d is given, the change in altitude in minutes of arc corresponding to a change of one degree in declination; a multiplication table on the inside of the back cover and on the page facing it is offered for purposes of interpolation.

This second half of each volume allows one to reduce observations of the Sun, Moon, planets and 13 additional bright stars whose declinations are numerically less than 28° and for which data are available in the Air Almanac. In both sections of the volume, data for north latitudes are printed on one pair of pages, for south latitudes on the following pair of pages. At the top of each page is a clear indication in heavy, black type of the hemisphere to which the data apply.

One of the features which makes for speed in the use of these tables is that the tabulated altitudes are already corrected for refraction under normal conditions at an elevation of 5000 feet above sea level; thus an interpolated altitude is directly comparable with an altitude measured by bubble octant from a plane flying at 5000 feet. A small table of additional corrections due to refraction is given for elevations other than 5000.

The bulk of the fourteen volumes, their weight and the fact that they are useful only between 70° North and 70° South are their chief disadvantages. The rapidity and simplicity of their use recommends them for the relatively crowded quarters of a plane under the perhaps unfavorable conditions of light, cold, low oxygen content of air, etc.

The excellent design of the tables indicates that experienced table-makers were consulted. This shows in the use of critical tables, the placing of the various tables, choice of type, and in other ways.

For purposes of comparison, the following table is offered:

	vol. (cc.)	weight (lbs.)	no. entries	no. nos. taken out	no. addits. and subtrs.	time reqd. (secs.)	cost	useful in latitudes	min. alt.
H. O. 208	317	0.8	4	8	4	75	$1.20	90°S to 90°N	−90°
H. O. 211	211	0.5	7	9	5	90	0.90	90°S to 90°N	−90°
H. O. 214	9752	15	3	4	1	50	18.00	80°S to 80°N	+5°
H. O. 218	11900	19	3	4	1	40	—	70°S to 70°N	+10°

No great emphasis is to be placed on the estimates of time required for the use of the tables in the standard way; they represent only the times required by a few experts working under the most favorable conditions. The number of entries is taken as the number of different pages which must be consulted and this is increased by one if a volume has to be chosen among several. The important fact is that none of the methods requires as much time as does the observation of a celestial body.

C. H. SMILEY

The U. S. Hydrographic Office made a large contribution to the accuracy of the contents of H. O. no. 218, since it supplied the British, for comparison with their own computations, with all values for both altitude and azimuth to tenths, from the equator to the poles. —EDITOR.

107[C, D].—COAST AND GEODETIC SURVEY, *Table of S and T Values*, [Washington, D. C.], U. S. Govt. Printing Office, 1934, 45 p. 20 × 26.1 cm. Off-set printing of type script. Not available for public distribution.

Absolutely nothing but the tables are printed in this volume; $S = \log \sin x - \log x$, and $T = \log \tan x - \log x$. Here, as well as in the volumes of Peters, and others, the authors use column headings S and T, when $S + 10$, and $T + 10$ would be exact. This is a table of $S + 10$ and $T + 10$, 0°–4°30′, for every 10″, to 10D, with differences. Each page is devoted to 6′. These tabular values are based on the second as unit. There is no suggestion

as to how this table was prepared but it may have been partly an abridgement of Andoyer's table to 14D, with differences, 0°–3°, p. 581–599 of

(1s). H. ANDOYER, *Nouvelles Tables Trigonométriques Fondamentales* (*Logarithmes*), Paris, 1911.

A similar table to 8D, for each second, 0°–5°, 85°–90°, is also to be found in

(2s). J. BAUSCHINGER and J. PETERS, *Logarithmic-Trigonometrical Tables with Eight Decimal Places*, v. 2, Leipzig, 1911, p. 2–151. Another like table, to 7D, for each 10″, 0°–2°46′30″, is given in

(3s). G. VEGA, *Logarithmic Tables of Numbers and Trigonometrical Functions, translated from the fortieth edition of Dr. Bremiker's thoroughly revised and enlarged edition* by W. L. Fischer, 72nd ed., Berlin, 1890, p. 2–185. In

(4d). J. T. PETERS, *Hilfstafeln zur zehnstelligen Logarithmentafel*, Berlin, 1919, p. 25–26, are the values of $S + 10$, and $T + 10$, to 10D, for each .001°, 0°.000–2°.100. In

(5m). *Logarithmic and Trigonometric Tables*, ed. E. R. Hedrick, rev. ed., New York, Macmillan, 1935, p. 45, there are very brief tables for $S + 10$ and $T + 10$ (correctly marked) with the unit in minutes. There are other small tables of S and T, 0°–4.890°, with units in degrees, minutes, and radians, in

(6r). L. M. MILNE-THOMPSON and L. J. COMRIE, *Standard Four-Figure Mathematical Tables, Edition A*, London, MacMillan, 1931, p. 4–5.

Such are examples of four types of tables according as the units are seconds, degrees, minutes, or radians. For an angle of 2° the respective values of S are (1s). $\bar{6}.68557\ 48668\ 2354$; (4d). $\bar{2}.24178\ 91682$; (5m). $\bar{4}.46390$; (6r). $\bar{1}.9999$. Some further forms of S and T tables, in other units, may be mentioned.

(7c.s.). FRANCE, SERVICE GÉOGRAPHIQUE DE L'ARMÉE, *Tables des Logarithmes à huit Decimales des Nombres entiers et de* 1 *à* 120 000 *et des Sinus et Tangentes de dix Secondes d'Arc dans le Système de la Division Centésimale du Quadrant*. Paris, 1891. S and T are given for every centesimal minute up to 5^g, with centesimal seconds as units. $S = \bar{6}.19603\ 169$ ($2° =$ approx. 22222″).

(8mi.). FRANCE, SERVICE GEOG. DE L'ARMÉE, *Tables de Logarithmes à cinq Décimales pour les Nombres de* 1 *à* 12 000 *et pour les Lignes Trigonométriques dans le Système de la Division de la Circonférence en* 64 000 *parties égales* (*dixième du millième de l'artillerie*), Paris, 1916. On p. [229] there are tables of S and T for each 4 mils from 0 to 600, mil as unit. $S = \bar{5}.99191$ ($2° =$ approx. 356 mils).

(9mic.). J. DE MENDIZÁBEL TAMBORREL, *Tables des Logarithmes à huit Décimales des Nombres de* 1 *à* 125 000 *et des Fonctions Goniométriques sinus, tangente, cosinus et cotangente de centimiligone en centimiligone, et de microgone en microgone pour les* 25 000 *premiers microgones et avec sept décimales pour tous les autres microgones*, Paris, 1891, S and T, p. 1–59, for the first 12500 microgones, to 8D, unit microgone. $S = \bar{6}.79809\ 169$ ($2° =$ approx. 5555 mic.).

(10ci.). J. C. F. REY-PAILHADE, "Table des logarithmes à quatres décimales de toutes les lignes trigonométriques dans la division décimale du cercle entier," Soc. Géog. de Toulouse, *Bull.*, v. 19, 1900. S and T table, p. 99, to 4D for $x = 0.0(0.1)2.5$ cirs. $S = \bar{2}.7981$ ($2° =$ approx. .5555 cirs).

(11t.s.). N. HERZ, *Siebenstellige Logarithmen der trigonometrischen Functionen für jede Zeitsecunde, zum astronomischen Gebrauche herausgegeben*, Leipzig, 1885. S and T are given for each 1^s, to 7D, from 0^h0^m to 0^h20^m, the unit being a second. $S = \bar{5}.861\ 5779$ ($2° = 8^m = 480^s$).

Tables of S and T, defined as the negative of the ordinary S and T, are found in

(12d.). C. BREMIKER, *Logarithmisch-Trigonometrische Tafeln mit fünf Decimalstellen*, third stereotyped ed., edited by A. Kallius, Berlin, 1880, p. 180, where the table, to 6D, for every tenth of a degree, is from 0° to 4°, the degree as a unit. For 2°, S (Bremiker) = 1.758211.

S and *T* tables, with the sexagesimal second as unit, appeared already in the first stereotyped edition of

(13s). F. CALLET, *Tables Portatives de Logarithmes*, Paris, 1795. In his *Report . . . on Mathematical Tables*, London, 1873, J. W. L. Glaisher states (p. 54), "Tables of *S* and *T* are frequently called, after their inventor, Delambre's tables." In a letter of C. M. Merrifield written to Glaisher in 1873, listing matters he wishes to bring to his friend's attention, he notes "the so-called Delambre's tables of log (sin x/x) and log (x/tan x) really John Newton in 1658." I have examined Newton's *Trigonometria Britanica* (*sic*), of 1658, but as yet I have found no printed *S* or *T* tables before 1795. Delambre's dates are 1749–1822. We have already referred to the manuscript *S* and *T* tables of the *Tables du Cadastre* (*MTAC*, p. 34) possibly dating from 1792 or 1793.

S and *T* "are required for passing from log arc to log sin and log tan, and are of particular value in geodetic calculations, where long operations have sometimes to be performed with small arcs which are usually expressed in seconds, while four or five places of the second have to be retained" (3s).

R. C. A.

MATHEMATICAL TABLES—ERRATA

8. France, Service Géographique de l'Armée, *Tables des Logarithmes à huit Décimales des Nombres entiers de* 1 *a* 120000 *et des Sinus et Tangentes de dix Secondes en dix secondes d'Arc dans le Système de la Division Centésimale du Quadrant*. Paris, 1891. Compare *MTAC*, p. 36.

In the differences and proportional parts which correspond to Log cos $4^g75'$ to $5^g00'$,

	for					read			
	49	48	47	46		51	52	53	54
1	4.9	4.8	4.7	4.6	1	5.1	5.2	5.3	5.4
2	9.8	9.6	9.4	9.2	2	10.2	10.4	10.6	10.8
3	14.7	14.4	14.1	13.8	3	15.3	15.6	15.9	16.2
4	19.6	19.2	18.8	18.4	4	20.4	20.8	21.2	21.6
5	24.5	24.0	23.5	23.0	5	25.5	26.0	26.5	27.0
6	29.4	28.8	28.2	27.6	6	30.6	31.2	31.8	32.4
7	34.3	33.6	32.9	32.2	7	35.7	36.4	37.1	37.8
8	39.2	38.4	37.6	36.8	8	40.8	41.6	42.4	43.2
9	44.1	43.2	42.3	41.4	9	45.9	46.8	47.7	48.6

	for	read
Log Sin $4^g65'40''$	$\bar{2}.86355936$	$\bar{2}.86355935$
Log Tan 4 65 40	$\bar{2}.86472090$	$\bar{2}.86472089$
Log Cot 4 65 40	1.13527910	1.13527911
Log Cot 34 53 60	0.21981237	0.21981257
Log Cos 41 28 80	$\bar{1}.90143668$	$\bar{1}.90143666$

J. DE MENDIZÁBEL TAMBOREL, Sociedad Cientifica "Antonio Alzate," Mexico, *Revista*, v. 5, p. 9–10, 1891.

9. Authors of frequently used works in the field of Statistics display some carelessness in the preparation of tables they publish. Here are a few illustrations (an asterisk * denotes an exact result):

R. A. FISHER and F. YATES, *Statistical Tables for Biological, Agricultural and Medical Research*, Edinburgh, 1938. P. 33, $n_1 = n_2 = 2$, *for* 99.01, *read* 99.00*; and $n_1 = 2$, $n_2 = 3$, *for* 30.81, *read* 30.82. The same mistakes occur in

G. W. SNEDECOR, *Statistical Methods applied to Experiments in Agriculture and Biology*, Ames, Iowa, Collegiate Press, third ed., 1940, p. 184. On this same page (through $n_2 = 13$) are at least 53 other last figure errors of 1 to 3 units, which suggest that there may be 200 errors on the 4 pages of this table of 5% and 1% points for the *F* distribution. Five of these

53 errors occur also in FISHER and YATES. The careful worker will naturally hereafter turn to such tables as reviewed in RMT **102**.

F. E. CROXTON and D. J. COWDEN, *Applied General Statistics*, New York, Prentice Hall, 1939, p. 878, has the following errors:

	.05		.01		.001	
n_1	*for*	*read*	*for*	*read*	*for*	*read*
1			4999.0	4999.5*		
2	18.999	19.00*	99.008	99.00		
3			30.815	30.817		
4	6.945	6.944	18.001	18.000*	61.238	61.246

R. C. A.

Other errors in FISHER and YATES are as follows:

P. 15, l. 10, $A = \bar{y}$, not $\bar{x}$.
P. 28, footnote to table, the formula should read,

$$z\,(20 \text{ percent}) = \frac{0.8416}{\sqrt{h-1}} - 0.4514\left(\frac{1}{n_1} - \frac{1}{n_2}\right).$$

P. 42, Table XII, the entry for $p = 72$ should be 58.1 *not* 58.7.
P. 48, l. 1, solution 16, the letter *e* in block 2 is blurred; the block letters are *adefj*.

W. G. COCHRAN

Yet other slips in FISHER and YATES are as follows:

P. 8, l. 10, *for* "ordinate is $\frac{1}{2}\text{sech}^2 z$," *read* "ordinate is $\frac{1}{2}\text{sech}\,2z$."
P. 57, Table XXIII, $n = 39$, bottom of col. 2, *for* 496,388, *read* 4,496,388.

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10. LEO HUDSON, and E. S. MILLS, *Natural Trigonometric Functions Tables. Sine, Cosine, Tangent, Cotangent, Secant, and Cosecant to Eight Decimal Places. With Second Differences to ten Decimal Places, Semi-quadrantally arranged.* 1941; see RMT **80**.

The sines and cosines given in this table were checked against the values appearing in the Coast and Geodetic Survey Table (see RMT **77**). In the case of a discrepancy Peters' *Eight-figure Table* was referred to (see RMT **78**), and finally the function was calculated to fifteen places by using Peters' *Einundzwanzigstellige Werte der Funktionen Sinus und Cosinus* (Berlin, 1911). In this way 32 last-figure sine errors were found. One of these, at 0°02′, where the eighth figure should be increased by two units, was indicated on an errata slip in the volume. The other 31 errors were of a unit in the eighth place. The first 8 cases where the eighth digit should be diminished by 1 are listed below and then the 23 cases calling for an increase by unity.

(−1) 13°31′, 27°08′, 27°21′, 27°24′, 43°25′, 55°09′, 77°18′, 84°03′.
(+1) 1°15′, 2°10′, 2°25′, 8°43′, 9°35′, 11°39′, 11°42′, 12°45′, 18°32′, 33°00′, 34°36′, 38°49′, 39°53′, 40°59′, 42°17′, 51°08′, 54°55′, 60°33′, 67°05′, 67°25′, 71°05′, 80°48′, 88°24′.

In comparing the column "Diff. per second" with 1/60 of the differences per minute of eleven-place functions interpolated from Peters' 21-place values, it is noted that the last figure of the printed difference is totally unreliable; from 0° to 1° it is wrong in 27 cases; from 1° to 2°, it is wrong in 30 cases; and from 2° to 3° in 13 cases. It is obvious therefore that the sines and cosines of this table are not to be relied on for more than seven-place accuracy, especially after using these differences with linear interpolation. Computation to "ten decimal places" is wholly out of the question. In making a test with the thought of using this table for seven-place work instead of such tables as Benson (RMT **75**) or Ives (RMT **76**), it was found that, after setting up a routine, it is possible, when interpolating to hundredths of a second, to save nearly 25% of the time used in locating the function in Benson to the nearest ten seconds and then interpolating. Tangents and secants have not yet been checked.

It seems rather a shame that anyone should have put in the enormous amount of time and energy required to compute these values from a series, and not attain the accuracy that was already available in Peters' *Eight-figure Table*, 1939, or in an abridgement of Andoyer, 1916. It would not be much of a job to compute the differences per second to six significant figures instead of five, using ten-place functions, interpolated either from Peters or Andoyer or Pitiscus, which would make the table far more valuable than it is in its present state.

F. W. Hoffman,
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A comparison of Legendre's table of sines, to 15D, for each 15′ (*Traité des Fonctions Elliptiques*, v. 2, Paris, 1826, p. 252–255), readily revealed not only three of the errors noted by Mr. Hoffman, but also a similar error for 48°15′. Comparison with Andoyer's table of sines and cosines to 15D (*Nouvelles Tables Trigonométriques Fondamentales, Valeurs Naturelles*, v. 1, Paris, 1915) showed that for the sines the eighth place values should also be increased by unity at 10°31′ and 32°36′. In the C.G.S. table (which Mr. Hoffman used for comparison) there were also errors in the three new cases noted.

R. C. A. and D. H. L.

11. J. Y. Dreisonstok, *Navigation Tables for Mariners and Aviators* (H. O. no. 208), sixth ed., 1942; see RMT 103.

Tables I and IA of this volume have been recomputed at the Ladd Observatory, using 7-place logarithms and punched cards in Hollerith Machines. The comparison between values given in H. O. 208 and the newly computed values is complete only for *A* and *C*.

In table I, 1858 errata were found in *A*, of which 158 were of two or more units in the last place given. In table IA, 426 errata in *A* were noted, 9 of two or more units. 345 errata in C were located in table I, 8 of two or more units; 263 errata in *C* were found in table IA, 28 of two or more units in the last place given.

Thus a total of 2892 errata have been noted in *A* and *C*, 203 of which are of two or more units in the last place given. The largest error in *A* was 26 units in the last place; the largest in *C* 20 units. The largest error in a computed altitude resulting from one of these errata would be about 4.4 minutes of arc, with a corresponding error of position of 4.4 nautical miles. This largest error would probably not occur in ordinary navigation; it represents a theoretical maximum.

The list of 203 errata of two or more units in the last place are given below.

L	*t*	*A* should be	*L*	*t*	*A* should be	*L*	*t*	*A* should be	*L*	*t*	*A* should be	*L*	*t*	*A* should be
1°	75°	58608		89	140877	5°	73	51710		83	76521		82	68142
	77	64667	3°	66	38771		76	59129		84	79574		83	70469
	78	68066		67	40485		87	99328		87	87786		86	76717
	80	75821		77	63703		88	102764	8°	78	60465		88	79537
	81	80305		78	66934		89	105123		79	62953		89	80305
	84	97486		80	74200	6°	68°	41236		80	65517	10°	75	51086
	85	105123		82	82825		72	48863		81	68142		80	61311
	86	114329		83	87786		77	60742		84	76083		81	63426
	87	125836		84	93267		79	66424		86	80862		82	65517
	88	140877		85	99328		80	69493		87	82825		83	67554
	89	160767		89	125836		82	76083		88	84345	10°	84°	69493
2°	68	42481	4°	73	52304		83	79574	9°	64	33690		86	72876
	71	48514		75	57276		84	83144		65	35089		87	74200
	76	61211		76	59996		87	93267		70	42914		88	75199
	80	75199		79	69310		89	97486		71	44661		89	75821
	81	79537		80	72876	7°	65	35970		72	46475	11°	68	38270
	82	84345		81	76717		72	48144		77	56586		78	55378
	85	102764		82	80862		74	52360		78	58813		81	61096
	86	110812		88	110812		80	67554		79	61096		82	62953
	88	130680		89	114329		81	70469		80	63426		84	66424

L	t	A should be	L	t	A should be	L	t	A should be	L	t	C should be	L	t	C should be
	86	69310		83	52360	24°	87	38771	9°	86°	6	86	19	1644
12°	74	46792		88	55647	25°	81	35089	29	42	233	86	78	1166
	76	50169	17°	81	48357		83	35970	31	11	786	87	1	3039
	79	55378		85	51710	29°	85	30913	42	83	132	87	4	2438
	81	58813		86	52304	30°	74	25644	44	84	145	88	4	2614
	82	60465	18°	69	33719	38°	78	19580	55	18	751	88	5	2517
	87	66934		80	45546	42	65	13126	58	8	1132	89	1	3516
	89	68066		81	46475	52	78	9781	60	4	1457	89	3	3039
13°	69	38157		83	48144	66	76	3677	76	14	1233	89	4	2915
	81	56586		84	48863	67	16	253.3	79	15	1306	89	5	2818
	84	60742	18°	88°	50753	67	58	2526	79	17	1253	89	6	2739
	87	63703	19°	81	44661	71	59	1761	82	1	2615	89	8	2615
	89	64667		87	48240	71	78	2322	84	1	2739	89	10	2518
14°	78	50169		88	48514	73	62	1498	85	1	2818	89	15	2345
	82	55743	20°	74	36751	77	6	12.0	85	2	2517	89	20	2224
	85	59129		81	42914	77	86	1122	85	66	1099	89	58	1830
	86	59996	21°	72	33719	81	14	31.1	86	1	2915			
	88	61211		77	38157				86	2	2614			
15°	70	37708		83	42482				86	3	2438			
	78	48466		89	44522				86	17	1690			
	80	51086	22°	79	38270									
	86	57276		84	41236									
	89	58608		88	42481									
16°	70	36751	23°	85	39916									
	78	46792		87	40485									

C. H. Smiley

12[A, D, P].—Earle Buckingham, *Manual of Gear Design. Section one. Eight Place Tables of Angular Functions in Degrees and Hundredths of a Degree and Tables of Involute Functions, Radians, Gear Ratios, and Factors of Numbers.* New York, Machinery, 1935. 183 p. 21.2 × 27.9 cm. $2.50.

No explanation of any kind is given of the sources, construction or checking of these tables; letters to the author asking for information have been ignored. Hence a thorough examination was necessary in order that their value could be appraised.

Pages 8–97 give 8-figure values of sine, cosine, tangent and cotangent at interval 0°·01(=36″). This section has been compared by Mr. Sidney Johnston with every 36th value in Peters' *Achtstellige Tafel der trigonometrischen Funktionen für jede Sexagesimalsekunde des Quadranten*, Berlin, Reichsamt für Landesaufnahme, 1939. The corrections thus found were then confirmed by the present writer from Briggs' *Trigonometria Britannica* (1633), and afterwards analyzed to discover the mode of preparation.

All errors in the sines and cosines greater than 0·55 units in the eighth decimal are shown in Table I. The error is in units of the last decimal, in the sense True Value *minus* Buckingham. Of the remaining 96 end-figure errors, 7 are cases where Buckingham's value is too high, but only by a turn of the figure, 4 are too low by about 0·54 units, while the remaining 85 are too low by amounts that vary between 0·50 and an upper limit that increases steadily to 0·53 as sin x increases from 0 to 1. The distribution of errors in sin x at intervals of 30° is shown alongside, treating cosines as the sines of their complementary angles. Not one of the 96 end-figure errors occurs in an angle that is a multiple of 0°·05. A comparison with Gifford's *Natural Sines to every Second of Arc and Eight Places of Decimals* shows that those tables have not been the source of the values before us; and obviously they have not come from the *Trigonometria Britannica*, which would have been by far the best source.

x°	no.
0	
	15
30	
	23
60	
	47
90	
	85

The explanation is that a table at interval 10″ has been used. This yields values at interval 0°·05 directly, while the remaining values have been formed by *linear* interpolation between the appropriate 10″ values. The maximum effect of neglecting second differences

(in sines) is $0 \cdot 03 \sin x$ units of the eighth decimal and so varies from 0 at $x = 0°$ to $0 \cdot 03$ at $x = 90°$—precisely what we found after eliminating 23 (Table I) + 7 + 4 = 34 values that appear to be attributable to lack of care in handling end figures. This accounts also for the observed increasing frequency as x increases. The only natural tables at interval 10″ are the *Opus Palatinum* of Rheticus (1596), the *Thesaurus Mathematicus* of Pitiscus (1613, but computed by Rheticus), and Andoyer's *Nouvelles Tables Trigonométriques Fondamentales*, Paris, Hermann, 1915. Had the former been used, it is certain that there would have been many more errors, as every table based on Rheticus contains errors that can be traced to his tables. Actually none of the errors in Tables I and II are due to the *Opus Palatinum*, which, in all the multiples of 10″ bordering the values in these two lists, never differs from Andoyer by more than one unit in the tenth decimal. We may take it, therefore, that Andoyer has been used.

There is only one serious error in the tangents, namely on page 20, tan 6°·40, where for 0·11226 797 we must read 0·11216 797. Twelve end-figure errors greater than 0·60 units of the eighth decimal are given in Table II. Besides these there are 126 cases in which the error does not exceed 0·60 units, and so may be considered negligible in computations. In four of these Buckingham's value is too low, while in the remaining 122 (analyzed alongside) it is too high. Here again the total number of errors, and their increasing frequency as x increases, correspond as nearly as possible to our expectation if 10-figure values at interval 10″ were interpolated linearly. Three of these errors occur where x is a multiple of 0°·05, namely 2°·20, 16°·50 and 16°·55. An examination of Andoyer showed that these were three of the nine cases in which the ninth and tenth decimals are 50. In eight of these the eighth decimal has been rounded up—in five cases correctly, but in the three cases under review incorrectly. In the ninth case (23°·50) the eighth decimal has been correctly rounded down. In all nine cases, the *Opus Palatinum* values also end in 50.

x °	no.
0	
	1
5	
	3
10	
	6
15	
	8
20	
	5
21	
	13
30	
	17
35	
	32
40	
	37
45	
	122

The cotangents present an interesting problem to the "error-analyst." Table III lists 27 cases in which the error is greater than a unit of the last decimal. The table alongside gives an analysis of errors not exceeding ±1·6 units. The reason for the break at 5°·71 is that the number of decimals increases at that point from 6 to 7. The reason for the second break will appear later. It will be realized that errors of 0·5 include those from 0·50 to 0·55 only. Bearing this in mind, the distributions (taking positive and negative frequencies separately) are approximately gaussian. After 18°·50 Peters gives 8 decimals, and Buckingham 7, so this portion was also read against Briggs-Gellibrand in order to detect errors between 0·50 and 0·55 units. In no case is a cotangent that is a multiple of 0°·05 in error, even by a turn of the figure.

Error	0°·00 to 5°·71	5°·72 to 17°·00	17°·00 to 45°·00
$\geq +1 \cdot 0$		16	
$+0 \cdot 9$	1	8	
$+0 \cdot 8$	2	11	
$+0 \cdot 7$	1	17	1
$+0 \cdot 6$	5	32	1
$+0 \cdot 5$	15	29	2
$-0 \cdot 5$	4	9	64
$-0 \cdot 6$	3	9	33
$-0 \cdot 7$	1	1	2
$-0 \cdot 8$		1	1
$-0 \cdot 9$		4	
$\geq -1 \cdot 0$	3	2	1
	35	139	105

The five errors marked with an asterisk could all have been easily detected by writing second differences, since first differences are already given in the tables. There can be no excuse for the neglect of this simple and elementary table-maker's precaution. It appears probable that the first three of these errors have arisen from confusion in copying. Thus:

2°·27 227227 has been copied for 227224
2°·29 006670 has been copied from 00666696
3°·24 665099 has been copied from 66502899

We are faced then with the fact that up to 17° Buckingham's tendency is to be too low, and from that point too high. The sudden switch-over at 17° is even more apparent from the full list of errors than it is from the summary given above. The table below gives observed and theoretical maxima and frequencies.

x °	Maximum obs.	comp.	obs.	No. in range comp.	o–c
17					
	0·79	0·74	27	34	−7
20					
	0·66	0·63	27	32	−5
25					
	0·54	0·57	21	17	+4
30					
	0·53	0·54	13	10	+3
35					
	0·52	0·52	7	7	0
40					
	0·51	0·52	5	5	0
45					
			100	105	−5

The theoretical maximum error resulting from linear interpolation of values at interval 10″ is, in units of the seventh decimal,

$$0\cdot 50 + 2 \cot x \operatorname{cosec}^2 x \operatorname{arc}^2 10'' \times 0\cdot 12 \times 10^7$$

in which 0·12 is the coefficient of the second difference for 0·4 and 0·6. The computed frequency is found with the aid of the difference Δ' (taken positively) of $\operatorname{cosec}^2 x$, and is

$$\Delta'(\operatorname{cosec}^2 x)10^7 \operatorname{arc}^2 10'' \times 0\cdot 1 \times 80 \times 57\cdot 3 = 10\cdot 8\,\Delta'(\operatorname{cosec}^2 x)$$

where

0·1 is average coefficient of second difference
80 is number of interpolates per degree.

The agreement between observation and theory supplies ample confirmation of the hypothesis of Buckingham's use of Andoyer and linear interpolation. Actually this section of the table is by far the most accurate, and has quite likely been done by a different computer.

It remains to account for the predominantly *low* values up to 17°. They have evidently been formed, not by interpolating Andoyer, but by taking the reciprocals of tangents formed by linear interpolation. These tangents would tend to be too *high* (as we found) and their reciprocals too *low*. The fact that a small number are slightly too high may possibly be the result of rounding off the tangents to nine decimals before reciprocating. The maximum error of such a procedure would be, in units of the seventh decimal, $0\cdot 006 \cot x \pm 0\cdot 005 \cot^2 x$, apart from any error arising from the final rounding off, i.e., a possible $\pm 0\cdot 5$. This gives errors up to $+1\cdot 1$ and $-0\cdot 9$ at 5°·7, $+0\cdot 70$ and $-0\cdot 62$ at 10°, and $+0\cdot 57$ and $-0\cdot 53$ at 17°. Actually the observed errors exceed these limits.

In six cases up to 17° the tangents and cotangents of the same angle are in error. In each case the errors are of opposite sign, the most striking examples being at 4°·59, 14°·88 and 14°·92.

One interesting fact emerges. In Andoyer's tangents the sixth decimal is one unit too low at 12° 15′ 10″, 20″, 30″ and 40″, but Buckingham is correct at 12°·26. Similarly the sixth decimal of Andoyer's value of cot 40° 43′ 20″ is a unit too high, but Buckingham is correct at 40°·72. The errors would, of course, be evident from the differences, since Buckingham's tables are linear at these points.

The above somewhat lengthy analysis affords an excellent example of the way in which errors in tables give clues to the sources from which the tables are derived, and the methods used in computing them. It is, however, far better that the author himself should give this information.

Page 98 is a table for converting minutes into decimals of a degree. In every case where the end figure is 6, it should be 7.

Pages 100–129 give the function involute x or $\tan x - x$ at interval 0°·01 up to 60°· 12 decimals are given up to 0°·5, 10 to 1°, 8 to 37°, and 7 to 60°. This table was examined by Mr. S. Johnston, by using the relation $\tan x - \operatorname{inv} x = x$, a process that would not detect errors of a unit or less in the last decimal. Some further examination was also made by Mr. J. C. P. Miller. Apart from a trivial omission of leading figures at 6°·03, there are two errors:

Page 104 inv 9°·15 for 6160 read 7160
Page 106 inv 13°·01 for 8468 read 8470

Pages 132–146 give, to 8 decimals, the radian equivalent of 0°(0°·01)45°. It should, of course, have been formed by taking multiples of $1° = 0^r\cdot 01745\ 32925\ 19943$. Actually multiples of 0·01745 329 were first taken up to 18°. At this stage comparison with $\pi/10$

showed a defect of just over $4\frac{1}{2}$ units in the eighth decimal. Instead of tracing the cause, and correcting it, the values at 17°·97, 17°·98, 17°·99 and 18°·00 were "fudged" by increasing them by 1, 2, 3 and 4 units respectively in the last decimal, to prevent a sudden discontinuity in the differences. Thereafter increments of 0·00017 45329 were again added, up to the end of the table at 45°, where the error has risen to more than seven units. Thus the value given for 45° is 0·78539 809, whereas $\pi/4$ is 0·78539 81634. It would be difficult to match this example of incompetent table-making.

Pages 148–169, described as "Brocot's Tables of Gear Ratios," give, to eight decimals, the values of all proper fractions (in their lowest terms) whose denominators do not exceed 120. Table IV gives a list of 27 errors found by Mr. S. Johnston and by Mr. J. C. P. Miller, who made a partial examination. Eight of these are of a unit in the last figure, and are thus of no engineering consequence. Every one of these errors and omissions could easily have been detected by the simple process of seeing that N/D and $(D - N)/D$ added precisely to 1. It was observed that, with one exception (argument 44/87 for 44/47), D is always greater than 60 in the error list. As *Machinery's Handbook* gives a similar table, also called Brocot's, but with D not exceeding 60, the inference is that Buckingham computed the values for D greater than 60, and introduced the errors now found. Compare RMT **87**.

Pages 172–183 give all the factors of *all* numbers up to 6009. A comparison by Mr. S. Johnston with the Br. Ass. Adv. Sci., *Mathematical Tables*, v. 5. (London, 1935) showed that two numbers on page 182 given as primes are really composite, namely 5183 = 71.73 and 5461 = 43.127. These errors do not occur in any other table that I know.

TABLE I—Sines and Cosines

page	col.	x°	for	read	error
8	sin	0·02	906	907	+0·6
8	cos	0·04	975	976	+0·6
10	cos	1·43	857	856	−1·0
11	sin	1·54	484	483	−0·6
15	cos	3·64	264	265	+0·9
20	cos	6·27	827	828	+0·6
21	cos	6·71	026	027	+1·2
22	cos	7·46	576	574	−1·6
25	sin	8·52	462	463	+1·4
26	sin	9·04	397	396	−0·7
27	sin	9·96	060	061	+1·0
34	sin	13·36	871	872	+1·1
42	sin	17·06	299	298	−0·6
47	cos	19·76	704	702	−1·7
54	sin	23·25	387	386	−1·4
79	cos	35·87	856	855	−0·7
80	cos	36·31	495	494	−0·6
81	cos	36·56	353	352	−0·7
85	sin	38·99	475	474	−0·6
89	cos	40·77	709	708	−0·8
89	cos	40·87	618	619	+0·7
92	sin	42·11	610	611	+0·8
97	sin	44·76	867	866	−0·6

TABLE II—Tangents

page	x°	for	read	error
12	2·27	971	972	+0·6
17	4·59	242	243	+0·9
29	10·59	428	427	−0·6
37	14·88	569	568	−0·8
37	14·92	323	324	+1·0
45	18·83	216	215	−0·8
45	18·84	700	699	−0·7
62	27·49	523	524	+1·1
75	33·79	894	893	−0·9
81	36·98	692	691	−0·7
85	38·82	553	552	−0·7
87	39·66	736	735	−0·6

TABLE III—Cotangents

page	x°	for	read	error
12	2·07	078	077	−1·3
12	2·27	227	224	−3·2*
12	2·29	670	667	−3·0*
14	3·24	099	029	−7·0*
14	3·42	251	250	−1·0
15	3·51	153	158	+5*
17	4·59	027	026	−1·3
21	6·66	885	886	+1·4
21	6·74	093	095	+2·4*
21	6·87	905	906	+1·2
23	7·88	360	361	+1·1
23	7·89	621	622	+1·0
24	8·31	853	854	+1·2
24	8·39	749	750	+1·0
25	8·64	140	141	+1·1
26	9·27	493	495	+1·6
26	9·28	305	307	+1·6
26	9·43	438	437	−1·4
28	11·19	992	993	+1·4
31	11·82	967	968	+1·0
34	13·24	731	730	−1·2
34	13·41	132	133	+1·1
35	13·52	037	038	+1·1
37	14·88	626	627	+1·2
38	15·36	019	020	+1·2
40	16·18	125	126	+1·0
94	43·36	504	503	−1·4

* These values, and the corresponding differences, should be corrected, even if the remaining errors are considered negligible.

TABLE IV—Brocot's Tables of Gear Ratios

page	N	D	for	read	authority
148	3	106	·02830 187	··· 189	J
148	3	86	Omitted	·03488 372	M; J
148	3	70	Omitted	·04285 714	M; J
149	7	79	·08860 760	··· 759	J
150	9	67	·13432 856	··· 836	J
151	13	72	·18055 555	··· 556	J
152	22	101	·21789 178	·21782 178	J
153	28	103	·27184 465	··· 466	J
154	32	113	·28318 585	··· 584	J
155	38	107	·35513 919	·35514 019	J
156	33	85	·38823 530	··· 529	J
156	46	117	·39316 293	··· 239	J
156	41	[100]	$D = 00$	$D = 100$	M
157	41	97	·42268 042	··· 041	J
157	46	103	Omitted	·44660 194	M; J
157	31	72	·43055 555	··· 556	J
157	39	79	·49367 087	··· 089	J
157	50	101	·49504 951	··· 950	J
159	41	79	Omitted	·51898 734	M; J
161	67	105	·63803 524	·63809 524	J
162	59	89	·66291 135	·66292 135	J
165	62	79	·78481 083	··· 013	J
166	61	73	·83561 484	··· 644	J
166	98	117	·83760 601	··· 684	J
167	95	[119]	$D = 119$	$D = 109$	M
168	44	[87]	$D = 87$	$D = 47$	M
168	107	112	·95535 710	··· 714	J

L. J. C.

In *Math. Gazette*, v. 26, Dec. 1942, p. 226–230, J. C. P. Miller has an article entitled "The decimal subdivision of the degree," which is also a review of Buckingham's book. Many of the facts stated above were first published in this article; for example, besides the 7 errors credited to M in Table IV, 12 more of the others were also published in his own review. —EDITOR.

UNPUBLISHED MATHEMATICAL TABLES

In *MTAC*, p. 27, we referred to an unpublished ms. of the late A. J. C. CUNNINGHAM giving the complete factorization of $n^2 + 1$ for $1 \leqq n \leqq 15{,}000$. Through L. J. Comrie we were informed by a letter, dated 5 May 1943, from A. E. Western, custodian of the Cunningham mss. of the London Mathematical Society, that this ms., as well as others, and many of the Society's books, housed in the library of University College, London, were destroyed by an enemy air raid.

4[L].—PROJECT FOR COMPUTATION OF MATHEMATICAL TABLES, *Spherical Bessel Functions*. Ms. in possession of the Project.

The Spherical Bessel Functions

$$Q_n(x) = \sqrt{\frac{\pi}{2x}}\, J_{n+\frac{1}{2}}(x)$$

occur in a wide variety of problems of wave motion, potential theory, heat conduction and quantum mechanics. The Project's preliminary manuscript is of the functions $Q_n(x)$ for $n = 0, \pm 1, \pm 2, \cdots, \pm 21$ and $x = [0(0.01)10;\ 8S\text{–}10S]$, with second and fourth central differences. It is contemplated to extend this table for values of n ranging from -20 to -35, $n = 20$ to $n = 35$ and for $x = [10(0.1)30;$ about $7S]$.

Tables related to $Q_n(x)$ are contained in the Br. Ass. Adv. Sci., *Reports*, for 1907, 1909, 1914, 1916, 1922 and 1925. In these reports the tabulated functions are given for $x = 0.1(0.1)2, 1(1)20, 10(10)100, 100(100)1000$ for various values of n ranging from -22 to 44. Most of the entries are given either to 7S or to 12D. Compare *MTAC*, p. 70–73.

Hayashi's *Tafeln der Besellschen, Theta-, Kugel- und anderer Funktionen* (Berlin, Springer, 1930) contains tables of $J_{\pm 1/2}$ and $J_{\pm 3/2}$ with most entries given to 12D. An abridged table of these functions is reproduced in his *Fünfstellige Funktionentafeln* (Berlin, Springer, 1930) as well as in Jahnke and Emde's *Funktionentafeln mit Formeln und Kurven*, third rev. ed., Leipzig, Teubner, 1938.

Lommel's big article, Bay. Akad. Wiss., *Math. Natur. Abt., Abh.*, v. 15, 1886, p. 531–663, contains tables of $J_{n+1/2}$ to six decimals for integral arguments up to 50, with n depending upon the argument. Part of this table is reproduced with considerable additions in G. N. Watson's *Treatise on the Theory of Bessel Functions*, Cambridge, 1922.

With respect to the interval of the argument (.01), the number of significant figures in the entries and the range of the order n, the table of $Q_n(x)$ reported here supersedes existing tables.

A. N. Lowan

5[L].—Project for Computation of Mathematical Tables, *Bessel Functions of Fractional Order*. Ms. in possession of the Project.

Bessel Functions $J_\nu(x)$ and $I_\nu(x)$ for the fractional orders $\nu = \pm \frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{4}$ and $\pm\frac{3}{4}$ are encountered in many problems of electrical engineering pertaining to problems of propagation and diffraction of electromagnetic waves. The Project's preliminary manuscript is of these functions for $x = [0(0.01)25; 10D]$. The manuscript contains also the values of $P(x)$ and $Q(x)$ in the asymptotic expansion of these functions in the form

$$P(x) \cos (x - \tfrac{1}{2}\nu\pi - \tfrac{1}{4}\pi) + Q(x) \sin (x - \tfrac{1}{2}\nu\pi - \tfrac{1}{4}\pi)$$

for x ranging from 25 to about 30,000, at various intervals depending on the ease of interpolation, to 10D. The manuscript is also available for the functions $I_\nu(x)$ where $\nu = \frac{1}{3}, \frac{2}{3}, \frac{1}{4}$ and $\frac{3}{4}$ and $x = [0(0.01)10; 10D]$, $x = [10(0.01)25; 10S]$ as well as for the functions $I_{-\nu}(x)$ where $\nu = \frac{1}{3}, \frac{2}{3}, \frac{1}{4}$ and $\frac{3}{4}$ and $x = [0(0.01)10; 10D]$, $x = [0(0.01)13; 10S]$ and values of $e^{-x}I_{\pm\nu}$ to 10D, obtained from the asymptotic expansion. Modified central differences of various orders are included in the manuscript. In addition to the above, thirteen-place values of the functions $J_{\pm\nu}(x)$ and $I_\nu(x)$ are available for the integral arguments 10, 11 $\cdots$ 25. For these arguments the first fifteen derivatives have also been computed to a number of places adequate for the subsequent subtabulation of the functions to about 12D. For the arguments $x = 10, 11, 12$, and 13, the values of $I_{-\nu}(x)$ were computed to 13S. For these arguments the first fifteen derivatives have also been computed to a number of places adequate for the subsequent subtabulation of the functions to about 12S.

Bessel functions of various fractional orders are given by Dinnik, Bursian and Karas. The reader is referred to RMT **11**, **59**, and **72**. Additional tabular material on these functions is contained in the article by G. N. Watson in the R. So. London, *Proc.*, v. 94A, 1918, p. 204. Some of the existing tables of J_ν go up to $x = 8$; others up to about 15, at intervals of 0.1, the entries being given mostly to 4S.

With respect to the range and interval of the argument and the number of significant figures, the tables of $J_\nu(x)$ recorded here, supersede practically all existing tables. Except for Bursian and Dinnik's short tables of I_ν, the tables of $I_\nu(x)$ reported here are almost all new. The values of $P(x)$ and $Q(x)$ in the asymptotic expressions above mentioned, are entirely new.

A. N. Lowan

6[I, K].—*Tables of Lagrangian interpolation coefficients*, Preliminary Ms. prepared by, and in possession of the PROJECT FOR COMPUTATION OF MATHEMATICAL TABLES, 50 Church St., New York City.

The ranges and intervals of the various tables included in the manuscript are as follows:

Three-point interpolants,	$x = [-1(.0001)1]$
Four-point interpolants,	$x = [-1(.001)0(.0001)1(.001)2]$
Five-point interpolants,	$x = [-2(.001)2]$
Six-point interpolants,	$x = [-2(.01)0(.001)1(.01)3]$
Seven-point interpolants,	$x = [-3(.01)-1(.001)1(.01)3]$
Eight-point interpolants,	$x = [-3(.1)0(.001)1(.1)4]$
Nine-point interpolants,	$x = [-4(.1)4]$
Ten-point interpolants,	$x = [-4(.1)5]$
Eleven-point interpolants,	$x = [-5(.1)5]$.

The tabulated entries are either exact or given to 10D. Interpolants make it possible to perform interpolation using the actual entries in the table without recourse to tabulated differences. (It should be noted that n-point interpolants correspond to approximation by a polynomial of degree $n - 1$.)

The only other printed tables of interpolants available in the literature are:

(1). E. V. HUNTINGTON, "Tables of Lagrangian coefficients for interpolation without differences,"[1] Am. Acad. Arts & Sci., *Proc.*, v. 63, 1929, p. 421–437; this table lists four-point and six-point interpolants at intervals of .01, exact values or 8D, for interpolating in the various parts of a table. It contains also a short table for cumulative subtabulation to 1/5th of the interval.

(2). T. L. KELLEY, *The Kelley Statistical Tables*, New York, Macmillan, 1938; these tables list four-point interpolants at intervals of .001 and six-point interpolants at intervals of .01 to 10D, as well as a few eight-point interpolants to 11D.

(3). ORDNANCE DEPARTMENT, *Table of Lagrangean Interpolation Coefficients*, Washington, D. C., June 1941. 40 p. 20.3 × 26.7 cm.; not available for public distribution. Five-point interpolants, prepared from the above-mentioned preliminary ten-place ms. table, for $x = [0(0.001)2.000; 7D]$. In September 1942 this was reprinted, for general distribution, by the Marchant Calculating Machine Co., Oakland, Cal., 23 p. 21.5 × 28.1 cm.

For a theoretical discussion of interpolants, the following references may be mentioned:

G. RUTLEDGE, "Fundamental table for Lagrangian coefficients," *Jour. Math. & Phys.*, M.I.T., v. 8, 1929, p. 1–12; G. RUTLEDGE and P. CROUT, "Tables and methods of extending tables for interpolation without differences," *ibid.*, v. 9, 1930, p. 166–180 (basic values for the computation of Lagrangean coefficients of various orders); and K. PEARSON, *On the Construction of Tables and on Interpolation, Part I* (*Tracts for Computers*, no. II), Cambridge, Univ. Press, 1920, p. 22–56. In this last we find short tables of coefficients in 4, 5, 6, 7, 8, 9, 10, and 11-point interpolants, with considerable expository material.

A. N. LOWAN

[1] A survey to investigate the reliability of these "Tables" will be the basis of a report. —EDITOR.

7[D, L].—*Table of* $\sin^{-1} x$. Preliminary Ms. prepared by, and in possession of, the PROJECT FOR COMPUTATION OF MATHEMATICAL TABLES, 50 Church St., New York City.

This is a table of $\sin^{-1} x$ for $x = [0(.001).99(.00001)1; 12D]$, with second and fourth central differences. The extent to which this table will supersede existing tables will be apparent from the following list of previous tables of $\sin^{-1} x$ for real values of x:

(1) J. M. PEIRCE. *Three and Four Place Table of Logarithmic and Trigonometric functions*. Boston, 1871; later reprints or editions, 1874, 1876; also *Mathematical Tables*, 1879; later

reprints or editions, 1880, 1886, 1891, 1896, and 1903; varying formats. These tables contain $\sin^{-1} x$ to hundredths of a degree for arguments $\log x$ at intervals of .01 or less.

(2) H. SCHUBERT. *Fünfstellige Tafeln und Gegentafeln für logarithmisches und trigonometrisches Rechnen.* Leipzig, 1897, p. 126–141. This table lists $\sin^{-1} x$ for x = [0(.001).89(.0001)-.999(.00001).9999(.000001)1; 5D].

(3) K. HAYASHI. *Sieben- und mehrstellige Tafeln der Kreis- und Hyperbelfunktionen und deren Produkte sowie der Gammafunktion,* Berlin, Springer, 1926. Includes $\sin^{-1} x$ for x = [0(.00001).001; 20D], [.001(.0001).0999; 10D], [.1(.001).999; 7D].

(4) K. HAYASHI. *Fünfstellige Funktionentafeln,* Berlin, Springer, 1930. Lists values of $\sin^{-1} x$ for x = [0(.01)1; 5D].

(5) L. M. MILNE-THOMSON and L. J. COMRIE. *Standard Four-Figure Mathematical Tables,* Editions *A* and *B*, London, MacMillan, 1931. Contains $\sin^{-1} x$ for x = [0(.001).99(.0001)1; 4S] in radians with first differences.

(6) R. A. DAVIS. *Table of Natural Sines and Radians.* Oakland, California, Marchant Calculating Machine Co., Nov. 1941, 8 p. 21.5 × 28 cm. This table lists $\sin^{-1} x$ for x = [0(.001)1; 6D], with differences.

Short tables of $\sin^{-1} x$ for complex values of x may be found in E. JAHNKE and F. EMDE, *Funktionentafeln mit Formeln und Kurven,* second ed., Leipzig, 1933, p. 68; and much longer ones in R. HAWELKA and F. EMDE, *Vierstellige Tafeln der Kreis- und Hyperbelfunktionen sowie ihrer Emkehrfunktionen im Komplexen,* Brunswick, 1931, p. 21–41.

The fundamental importance of tables of $\sin^{-1} x$ requires no elaboration.

A. N. LOWAN

8[A].—HERBERT ELLIS SALZER (1915–   ), *Tetrahedral Numbers.* Ms. of the first thousand tetrahedral numbers in possession of the author, and of the Brown University Library. 6 p.

The tetrahedral numbers $n(n+1)(n+2)/6$ or ${}_{(n+2)}C_3$ were calculated for $n = 1(1)1000$. For a discussion of their history and properties, see L. E. Dickson, *History of the Theory of Numbers,* v. 2, Washington, D. C., 1919, chap. 1. In addition to the theorems about tetrahedral numbers which are mentioned by Dickson, the author adds the following original empirical theorem: Every square of an integer is the sum of 4 (or less) positive tetrahedral numbers. (It has been verified for the first 200 squares.)

The tetrahedral numbers have these well-known properties: (1) The sum of the n^{th} and $(n+1)^{\text{th}}$ is equal to the sum of the squares of the first $(n+1)$ integers. (2) The difference between the n^{th} and $(n-1)^{\text{th}}$ entry is equal to the n^{th} triangular number, $n(n+1)/2$. (The first 20,000 triangular numbers were printed in E. de Joncourt's *De Natura et praeclaro usu simplicissimae speciei Numerorum Trigonalium,* 1762). (3) The square of the difference between the n^{th} and $(n-1)^{\text{th}}$ entry is equal to the sum of the first n cubes.

H. E. SALZER,
50 Church St., New York City

9[A].—EDWARD BRIND ESCOTT (1868–   ), *Amicable Numbers.* Ms. in possession of Mr. Escott and in the Library of Brown University.

This Ms. contains a list of the 391 Amicable Numbers discovered during the past 2500 years. Two numbers are called amicable if each equals the sum of the aliquot divisors of the other. Iamblichus attributes to Pythagoras (*c.*540 B.C.) the discovery of the first pair of amicable numbers 220 ($=2^2\cdot5\cdot11$) and 284 ($=2^2\cdot71$). The next two pairs were discovered by Fermat and Descartes. Euler added 59 pairs in the next century but even to the end of the nineteenth century only 66 pairs had been discovered. Details in this regard may be found in L. E. Dickson, *History of the Theory of Numbers,* v. 1, Washington, 1919, p. 38–50.

The complete record, of discoveries, with approximate dates, is as follows:

Pythagoras	1 (540 B.C.)	L. E. Dickson	2 (1911)
Fermat	1 (1636)	T. E. Mason[1]	14 (1921)
Descartes	1 (1638)	P. Poulet[2]	68 (1929)
Euler	59 (1747–50)	A. Gérardin[3]	5 (1929?)
Legendre	1 (1830)	E. B. Escott[4]	235 (1934)
B. N. I. Paganini	1 (1867)	B. H. Brown[5]	1 (1939)
P. Seelhoff	2 (1884)	Total (May 1943)	391

E. B. Escott,
1019 South East Ave., Oak Park, Ill.

[1] T. E. Mason, "On amicable numbers and their generalizations," *Amer. Math. Mo.*, v. 28, 1921, p. 195–200.
[2] P. Poulet, *La Chasse aux Nombres. Fascicule I. Parfait, Amiables et Extensions*, Brussels, 1929, p. 28–51. The 156 pairs of amicable numbers, known at this time, are here classified, and include the 68 new pairs found by Poulet.
[3] All of these pairs are in Poulet's list of 1929. It is possible that their discovery was announced earlier in Gérardin's periodical *Sphinx Oedipe.*
[4] P. Poulet, "De nouveaux amiables," *Sphinx*, v. 4, 1934, p. 134–135, Poulet here states that Mr. Escott had sent him 322 pairs of amicable numbers; he prints 21 pairs discovered by Mr. Escott, and all but one of the 42 numbers are less than 10^9. The other 214 pairs have not yet been published.
[5] B. H. Brown, "A new pair of amicable numbers," *Amer. Math. Mo.* v. 46, 1939, p. 345.

MECHANICAL AIDS TO COMPUTATION

5[X].—R. E. Beard, "The construction of a small-scale differential analyser and its application to the calculation of actuarial functions," Institute of Actuaries, *Jn.*, v. 71, 1942, p. 193–227 + 3 plates.

The author adopts the principles of the differential analyzer as conceived by Kelvin, Bush, and Hartree to construct another small-scale machine for experimental work in finding solutions of differential equations that arise in actuarial science. The article contains a description of its mechanical principles, operation, and application to actuarial functions. At the end is a detailed 16-page abstract of the discussion following the presentation of the author's paper.

The most important unit in the machine is the integrator where $(1/a)\int y\,dx$ is obtained from two mutually perpendicular wheels which touch each other. When the first wheel turns through angle dx, the rim of the second wheel (of radius a) at variable distance y from the center of the first wheel, will turn through distance ydx, corresponding to the angle ydx/a. Use is made of a torque amplifier to increase the friction; this is necessary for proper operation. Adequate description of the adding units, input and output tables and counting devices precedes the discussion of the operation of the machine.

The first stage in the operation consists of the drawing of a diagram showing the required units properly connected. Vannevar Bush's notation is followed. The reader can learn at a glance the notation for the integrator, input and output table, adding unit and right- or left-hand gears, after which he can understand all the diagrams. Initially the author gives a clear description of the set-up for simple integration and its application to the "circle test" of the machine, i.e., the integration of $y'' = -(1/k^2)y$. Finally a description of an elegant device to obtain the integral of a product without getting the product itself, completes the general introduction to the use of the analyzer.

The calculation of interest functions v^n and $(1+i)^n$ is reduced to $e^{-n\delta}$ and $e^{n\delta}$ or an equation of the form $dy = \dfrac{y}{C}dt$. One immediately obtains $\int f(t)v^t dt$ by integration of a product. For $f(t) = 1$, we get $\bar{a}_{\overline{t}|}$. For mortality calculations the machine solves $\dfrac{d}{dt}\,{}_tp_x = -\,{}_tp_x\mu_{x+t}$, where ${}_tp_x$ is the probability of survival. A detailed discussion of joint-life functions and

whole-life annuities is attended in the latter case with the operation procedure for $\frac{d\bar{a}_{x+t}}{dt} = (\mu_{x+t} + \delta)\bar{a}_{x+t} - 1$. Two methods are mentioned for contingent functions, one of which employs integration of a product and the other of which calculates $|_t\bar{A}'_{xy}$ directly. (The diagram for the latter method contains 10 units). Computation of policy values leads to the solution of

$$-\frac{d}{dt}\,{}_t\bar{V}_x = (\mu_{x+t} + \delta)(1 - {}_t\bar{V}_x) - \frac{1}{\bar{a}_x}$$

which is easily explained by a diagram of 4 units. For some of the simpler of these functions, comparison of the true values with the machine values shows that this analyzer is capable of accuracy to within several units in the third significant figure. In concluding, the author mentions possible extensions in the applications of the machine. He also emphasizes its advantages in that it calculates actuarial functions directly from the "observed functions" μ_x. At the end is a "refutation" of the three main objections to the use of his machine: length of time required, insufficient accuracy, and inconvenient form of μ_x. A few references are cited.

The ensuing discussion contains interesting historical information on similar differential analyzers. The members present furnished elaborate criticisms of the use and limitations of this machine. Most of these comments should be of particular interest to actuaries and also of general interest to anybody concerned with applications of this type of differential analyzer.

H. E. Salzer

NOTES

8. Henry Briggs and His Dates.—We have already referred to this great table-maker, and one of his works (p. 10, 13, 26, 33, 44). Even in the case of first-class authorities there is divergence of statement with regard to the years of his birth and death. D. E. Smith (*History of Mathematics*, v. 1, 1923) gives them correctly, as born February 1560–61 (1561 N. S.), died 26 January 1630/31 (1631, N. S.); the dates are also correctly given by W. W. R. Ball (*Short Account of the History of Mathematics*, 1912). Yet the *Dict. of Nat. Biography* (1886), and its concise one volume edition (1930), give his dates as 1561–1630. From the thirteenth century to 1753, in England and Ireland, the year began on March 25. Hence O. S. the dates of Briggs are 1560–1630, but N. S. 1561–1631. The account of Briggs in J. Ward, *The Lives of the Professors of Gresham College*, London, 1740, makes clear why this work, and such authorities as the following, give the year of Briggs's birth as 1556: J. C. Poggendorff, *Biographisch-Literarisches Handwörterbuch;* M. Cantor, *Vorlesungen über Geschichte d. Math.* v. 2; H. T. Davis, *Tables of the Higher Mathematical Functions*, v. 1, 1933, p. 32; F. Cajori, *A History of Mathematics;* J. Tropfke, *Geschichte der Elementar-Mathematik*, v. 1, 3rd ed., Berlin, 1930, p. 58; *Encyclopædia Britannica* (11th ed. 1910, 14th ed. 1929 and 1936). *DNB* (1886) first produced the authority for 1561.

Briggs was the author of tables for the improvement of navigation, published in the second edition (1610) of Edward Wright's *Certain Errors in Navigation detected and corrected*. Briggs's edition of six books of Euclid's Elements was published anonymously in 1620. His *Logarithmorum Chilias Prima*, "printed for the sake of his friends and hearers at Gresham College" (16 p., 1617) contained log N for $N = [1(1)1000; 14D]$,—the first table of logarithms to the base 10 ever computed or published. But Briggs, Napier,

and Kepler did not have our concept of logarithms as powers of a base. In 1624 appeared the great work containing log N for $N = [1(1)20\,000$ and $90\,000(1)\,101\,000; 14D]$; only some copies of these tables have the Chiliad for $N > 100\,000$. His remarkable Canon of Sines, and other material (published posthumously, 1633) has been already described. Note *MTAC*, p. 100.

R. C. A.

9. Cayley and Tabulation.—Except for a brief first paragraph which has nothing to do with the sequel, the following is a letter (in my possession) written to J. W. L. Glaisher (1848–1928) by Arthur Cayley (1821–1895):

"Dear Mr. Glaisher,

. . . .

"Apropos of tabulation—in many arithmetical operations you are sure of your last figure—i.e., the result as directly obtained to a given number of figures, say three, is accurate as far as it goes but always in defect e.g., .246. To correct the last figure you have to go *one* figure further; if the next figure is 0, 1, 2, 3 or 4, you leave the last figure unaltered—if it is 5, 6, 7, 8 or 9, you increase it: of course the maximum error is $\pm.0005$.

"Now suppose you were not to correct the last figure at all, but tabulate always in defect—*and in using the tables* supply in every case a last figure 5. e.g. read your tabular number .246 as .2465 the maximum error would be as before $\pm.0005$ you would have saved yourself the calculation of the additional figure (the 4th figure) in making the table.

"To counterbalance this you have to work with 4 instead of 3 figures.

"Which is best? It seems to me the accuracy is *absolutely* the same. In favor of the ordinary plan it may be said, that the table is a thing made once for all and that the labour of the calculation *does not signify*—(a view for me rather than for *you*).

"In favor of the other plan; it would be perhaps hundreds or thousands of years before all the tabular numbers come into use—so that even admitting that in each single case the labour of using the final figure 5 is equal to what would have been the labour of correcting the figure (and it is certainly much less). There would be on the whole (i.e., considering together the labor of computing the table and of using it) a saving of labour.

Believe me, yours very sincerely,

Cambridge, 9th July 1874. A. Cayley"

R. C. A.

QUERY

3. A Shortrede Table.—In *Catalogue of the Library of the Royal Astronomical Society*, London, 1886, is the following entry: "Table of log $\dfrac{\text{vers } P}{\sin 1''}$ to every second of time as far as 1^h00^m4to. Poona, 1842." There is no suggestion as to the author of the table. Mrs. Grace O. Savage, librarian of the U. S. Naval Observatory, kindly drew my attention to the fact that in R. Astron. So., *Mo. Notices*, v. 8, 1848, p. 160, there is a reference to a memoir "On a formula for reducing observations in azimuth of circumpolar stars near elongation, to the azimuth at the greatest elongation" by Robert Shortrede (1800–1868), who spent a number of years in India; see also *MTAC*, p. 42. The report concludes with the statement that a table of "log $\dfrac{\text{ver. sin}}{\sin 1''}$" [sic] "for all arcs up to 1^h is added to the memoir." This is evidently the Poona table. Where may a copy be seen in America?

R. C. A.

QUERIES—REPLIES

3. Tables to Many Places of Decimals (Q 1; QR 1, 2).—When are many-place values needed? By the engineer or physicist using tables for a specific numerical problem, practically never. But by the ***computer of new fundamental tables*** not infrequently. They may be invaluable, or even indispensable, where the nature of the computing process employed is such that many decimal places are lost in the working; from the old trouble of "differences of nearly equal quantities," and particularly in the use of recurrence formulae. Some years ago, in order to settle a question as to the possibility of interpolating for fractional values of the order n of the Bessel function $J_n(x)$ (considered as a continuous function of the order n for a fixed parameter x), I computed from series, to 7D, $J_n(0.1)$ and $J_n(1.0)$ for $n = 0(0.01)3$. This table can be interpolated with 6th differences. It can be extended for $n > 3$ by the recurrence formula

$$J_{n+1}(x) = (2n/x)J_n(x) - J_{n-1}(x),$$

but repeated use of this formula loses decimal places; notably so in the case $x = 0.1$ we lose at least one place at every application.

While the method is inappropriate for finding a single value, it is comparatively rapid for forming a whole table. If such a table were required up to $n = 20$ say, it would probably pay to compute the fundamental 20 or 30 values to 25 or 30 decimals, in order to make sure of accuracy in the 6th or 7th decimal after employing the recurrence formula twenty times. It would then be necessary to compute the Gamma function to "many places."

C. R. Cosens

Engineering Laboratory,
Cambridge Univ., England

4. Tables to Many Places of Decimals (Q 1; QR 1, 2, 3).—The evaluation of integrals by means of reduction formulae and series provides a further illustration in answer to this query. When an integral of type

$$\int_0^{\pi/4} f(\tan a)da$$

is evaluated by expanding $f(x)$ in a Maclaurin series, and integrating term by term, it is necessary to use accurate values of the integrals of this type in which $f(x) = x^n$. These integrals may be found by means of a well known reduction formula which reduces them to the cases $n = 0$ and $n = 1$. The first of these is $\frac{1}{4}\pi$ and so when n is large the value of π is needed to a large number of places for the value of the integral is equal to the difference between $\frac{1}{4}\pi$ and a quantity which is very nearly equal to $\frac{1}{4}\pi$. When $n = 1$ the integral depends on log 2 and so this number is also needed very accurately. Though the integral for a large value of n is small this integral may have a large coefficient when use is made of the Maclaurin expansion. There are innumerable cases in which the use of a reduction formula is at present the only practical means of computing an integral and so it is desirable that there should be very accurate tables of some of the functions needed for the expression of standard types of indefinite integral. In addition to the logarithm, the inverse tangent, inverse sine, inverse hyperbolic functions, inte-

gral sine and cosine, the incomplete gamma function, dilogarithm and various integrals of Bessel functions, are of frequent occurrence.

H. B.

Attention is directed to Mr. Bateman's article "Occasional need for very accurate logarithms," *Amer. Math. Mo.*, v. 32, 1925, p. 249.—EDITOR.

5. SCARCE MATHEMATICAL TABLES (Q 2).—Since this query was sent to the printer I find that three copies of item *E*, [H. GOODWYN], *A Table of Circles*, 1823, are listed in catalogues of Edinburgh libraries, namely:

(a) *Catalogue of the Printed Books in the Library of the Faculty of Advocates*, Edinburgh and London, v. 3, 1874;

(b) *Catalogue of the Crawford Library of the Royal Observatory*, Edinburgh, 1890;

(c) *Catalogue of the Printed Books in the Library of the University of Edinburgh*, Edinburgh, v. 2, 1921.

Mr. C. R. COSENS (see above) has reported to me that a fourth copy is in the Library of the University of Cambridge. This copy, together with the other Goodwyn publications, including those mentioned *MTAC* 1, p. 22, are preserved with the following letter: "Miss Catherine Goodwyn presents to the Library of the University of Cambridge a complete set of the works of her late father, Henry Goodwyn, Esq. of Blackheath, Kent." "Royal Hill, Greenwich, Sept. 16, 1831."

While I have not yet found a library with items *C* and *D* I did find a third enlarged and greatly improved edition of these items in the Library of Massachusetts Institute of Technology with the following title: *Hütte Hilfstafeln zur I. Verwandlung von echten Brüchen in Dezimalbrüche, II. Zerlegung der Zahlen bis* 10 000 *in Primfaktoren. Ein Hilfsbuch zur Ermittelung geeigneter Zähnezahlen für Räderübersetzungen. Herausgegeben vom Akademischen Verein Hütte E. V. Berlin.* Dritte neubearbeitete Auflage. Berlin, 1922, vi, 83 p. 11.9 × 18.6 cm. Table I, the "Brocot" table, occupies p. 23–62; and Table II, factor table, p. 63–83; see RMT **87**, p. 21–22. What is especially interesting about this edition is that both of the tables were thoroughly checked by J. T. PETERS.

R. C. A.

CORRIGENDA

P. 26, MTE **4**, the editor regrets that ll. 3–6 give a wrong impression in the following three respects: (1) it should have been stated that l. 3 was an error printed on a slip pasted in the fourth edition before distribution; (2) l. 4, *for* "first," read "third" (as indicated in RMT **82**); (3) after the correct information for ll. 4–5 had been supplied by L. J. C. his signature was forged by the editor.

P. 33, for ll. 11–15, substitute the following:

1450 in a Latin codex in Munich[1], compiled by Theodericus Ruffi. The idea of the centesimal division of the degree was mainly suggested to Henry Briggs as the appropriate unit for his wonderful "sine canon"[2] (see RMT **79**), by Vieta's *Calendarij Gregoriani*, Paris, 1600, folio 29 (*Opera Mathematica*, Leyden, 1646, p. 487.)

P. 39, l. 3, *for* Andoyer items, *read* Andoyer item

P. 45, l. 12, *for* $5.00, *read* $2.50

P. 56, *for* l. 7, *read* $\ln 71 = 2 \ln 2 + \ln 3 + (\ln 5 + \ln 7)/2 + S\,(1/10081)$

P. 57, l. 4, *for* i–xxviii, *read* I–XXVIII

P. 58, heading last column, "abbreviation of"

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CLASSIFICATION OF TABLES, AND SUBCOMMITTEES

A. Arithmetical Tables. Mathematical Constants
B. Tables of Powers
C. Logarithms
D. Circular Functions
E. Hyperbolic and Exponential Functions
  Professor DAVIS, *chairman*, Professor ELDER
  Professor KETCHUM, Doctor LOWAN

F. Theory of Numbers
  Professor LEHMER

G. Higher Algebra
  Professor LEHMER

H. Tables for the Numerical Solution of Equations
J. Summation of Series

I. Tables connected with Finite Differences. Interpolation
K. Statistical Tables
  Professor WILKS, *chairman*, Professor COCHRAN, Professor CRAIG
  Professor EISENHART, Doctor SHEWART

L. Higher Mathematical Functions
M. Integral Tables
  Professor BATEMAN

N. Interest and Investment
O. Actuarial Tables
  Mister ELSTON, *chairman*, Mister THOMPSON, Mister WILLIAMSON

P. Tables Relating to Engineering

Q. Astronomical Tables
  Doctor ECKERT, *chairman*, Doctor GOLDBERG, Miss KRAMPE

R. Geodetic Tables
S. Physical Tables
T. Critical Tables of Chemistry
U. Navigation Tables

Z. Calculating Machines and Mechanical Computation
  Doctor COMRIE, *chairman*, Professor CALDWELL, *vice-chairman*
  Professor LEHMER, Doctor MILLER, Doctor STIBITZ, Professor TRAVIS

EDITORIAL NOTICES

The addresses of all contributors to each issue of *MTAC* are given in that issue, those of the Committee being on cover 2. The use of initials only indicates a member of the executive committee.

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